UNIT 2 THEORY OF DEMAND: AN ALTERNATIVE APPROACH

Structure

2.0 OBJECTIVES

In this unit our objective is to explain the decision taken by the consumer to maximise utility subject to the budget constraint. For that purpose it adopts a different approach, popularly known as Revealed Preference Approach. Continuing we move on to consumers' attitude towards risk. Finally, indirect utility function and related theorems are taken for discussion. After going through the unit you will be able to:

- Determine the optimum choice of a consumer under revealed preference approach;
- Explain how the substitution effect is negative;
- Examine consumers' attitude towards risk;
- Evaluate the properties of indirect utility functions; and
- Understand the development of duality theorems.

2.1 INTRODUCTION

In the previous unit we discuss the consumer behaviour under indifference curve approach where it is assumed that the consumer possesses a utility function. The next most important theory that deals with consumer behaviour is the theory of revealed preference. This theory allows prediction of the consumer's behaviour without specification of an explicit utility function, provided that she conforms to some simple axioms. In addition, the existence and nature of her utility function can be deduced from her observed choices among commodity bundles.

In the next section, we move on to more realistic aspect of consumer behaviour namely consumer choice involving risk, where there is probability of every event. Finally, we present the indirect utility function and its related theorems.

2.2 THEORY OF REVEALED PREFERENCE

2.2.1 Introduction

If consumer's taste and preferences do not change, then observation of her market behaviour or, actual act of choice between the commodity sets reveals her preference. The above statement is the basis of the revealed preference theory.

In this section we will show that although revealed preference theory have no concept of utility function like indifference curve approach, it still preserves all the main results of the consumer behaviour theory. Just like in the indifference curve approach, substitution effect is negative in this theory and also consumer is free from money illusion i.e., the demand will remain unchanged if prices of all goods and money income change proportionately. Under this theory, the slope of the ordinary demand curve can take any algebrical sign, which is also consistent with the indifference curve approach.

Given a price income substitution, if a consumer buys a collection of goods A, rather than an available collection B, and if B is not more expensive than A, then A has been revealed preferred to B. Let $p_A = (p_{1A}, p_{2A}, \ldots, p_{nA})$ be the set of prices at which the individual buys collection $A = (x_{1A}, x_{2A}, \ldots, x_{nA})$ and spurns $B = (x_{1B}, x_{2B}, \ldots, x_{nB})$. Then A is revealed preferred to B, if it is at least as expensive as B at the prices p_A at which A is purchased, i.e., if the following holds,

$\sum_{i=1}^{n} p_{iA}x_{iA} \geq \sum_{i=1}^{n} p_{iA}x_{iB}$, where right hand sum represents the cost of collection B at the prices p_A at which in fact A was purchased.

In a two-commodity framework, price and income are given by (p_1, p_2, M) and represented by price line pp'. Any other point on pp', such as B is just as expensive as A because they both are on the same budget line. Every point such as B_1 below the price line represents smaller amounts of both commodities than do some points on pp'; such lower points are cheaper than A.

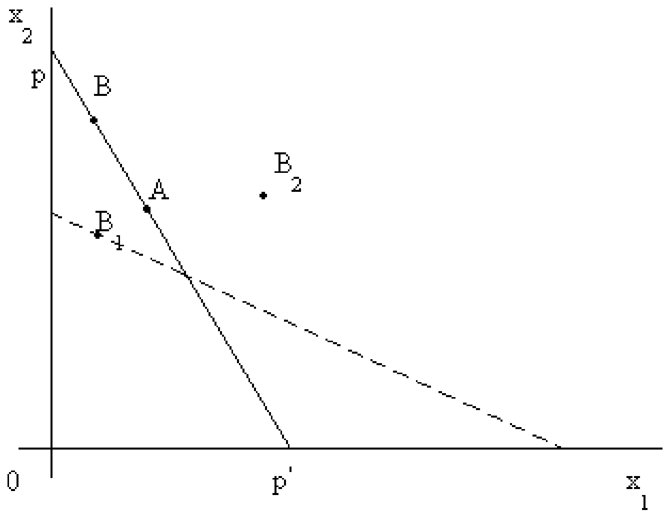

Fig. 2. 1: Concept of Revealed Preference

Therefore, because consumer bought A rather than any of these collections that were no more expensive, it follows that every point on or, below pp' is revealed inferior to A.

Any point above pp' is more expensive than A. None of such points like B_2, can be revealed inferior to A by consumer's purchase of A.

2.2.2 Axioms

Revealed preference theory is based on the axioms listed below.

- Consumer will spend all her income on goods. The consumer equilibrium always remains on the budget line.
- For any given price income situation, there corresponds a unique commodity bundle that the consumer chooses. Given the same income and prices, consumer will always choose the same bundle.
- For any given bundle, there is always a unique price income situation in which the consumer will be led to purchase that bundle.
- **Weak axiom of revealed preference:** If consumer chooses a bundle q^0 in some price income situation (p^0, M^0) and spurns q^1, where q^1 is not more expensive than q^0 at the prices at which q^0 is bought, then q^0 is revealed preferred to q^1. Then q^1 can never be revealed preferred to q^0 when q^0 is available. If in another price situation, consumer chooses the bundle q^1, then q^0 is not an available alternative at that price situation.
- Demand function is single valued in prices and income.

2.2.3 Derivation Of Ordinary Demand Function

Suppose, $p^0 = (p_1^0, p_2^0, \ldots, p_n^0)$, $q^0 = (Q_1^0, Q_2^0, \ldots, Q_n^0)^T$ and $q^1 = (Q_1^1, Q_2^1, \ldots, Q_n^1)^T$. Let M^0 be the money income and $p^0q^0 = M^0$ and $p^0q^0 \geq p^0q^1$, where p^0q^1 is the total expenditure of buying q^1 at p^0 set of prices. q^0 is revealed preferred to q^1. Let the set of prices when she buys q^1 be $p^1 = (p^1{}_1, p^1{}_2, \ldots, p^1{}_n)$, then q^0 is not an available alternative at p^1 price. $p^1q^1 < p^1q^0$ and $M^1 < p^1q^0$.

For simplicity lets consider a two-goods world and at initial prices and money income, budget line is AB, which is shown in the following figure.

According to the axiom, budget line is downward sloping and linear. Suppose the consumer chooses the bundle (x_1^0, x_2^0). Moreover suppose that for given money income M and price of the good two (viz. p_2) (i.e., given the intercept of the budget line) p_1 decreases. Then $\left(\frac{p_1}{p_2}\right)$ would fall. Now initial budget line is AB becomes flatter with same intercept. None of the commodity bundles on the new budget line are previously available. Therefore, according to weak axiom of revealed preference, consumer can choose any commodity bundle from the new budget line AC. Suppose it is at point V. That means ordinary demand curve can take any algebrical slope. In this case, x_1 increases due to fall in p_1 for given p_2 and M. Ordinary demand curve is downward sloping or, own price effect is negative.

Let us show that this own price effect consists of own substitution effect and income effect for a price change by using Slutsky's method, where real income is measured in terms of purchasing power.

Given the money income, as p_1 decreases, real income increases by which demand for x_1 changes. To ignore this, money income reduces proportionately so that real income in terms of purchasing power is constant i.e., after adjustment of money income, the budget line AC shifts parallely downward such that it passes through the original commodity bundle i.e., point z to maintain same purchasing power.

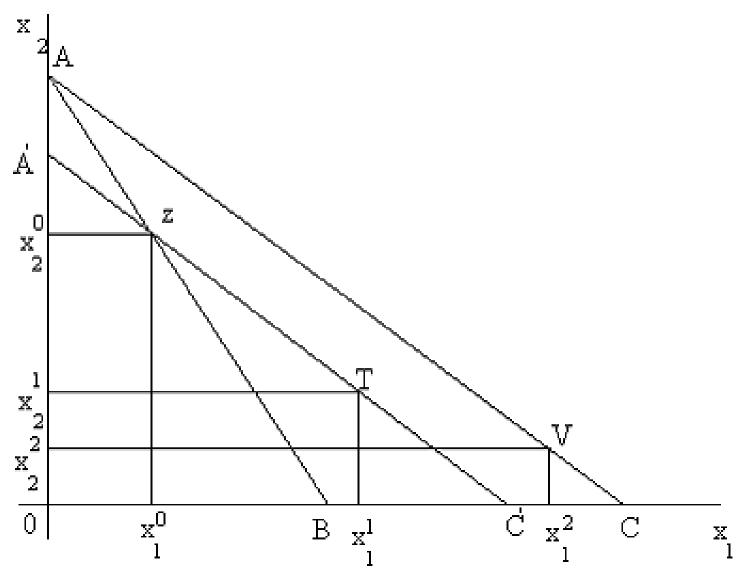

Fig. 2.3: Derivation of Consumer Demand Curve

Such a budget line is known as compensated budget line along which real income (in terms of purchasing power) is constant. This is denoted by line A'C' in the diagram. Note that the consumer always chooses a commodity bundle only from the compensated budget line A'C'.

But according to weak axiom of revealed preference, consumer can't choose any bundle between A'z since all these bundle are previously available at the budget line AB. But consumer doesn't prefer these as she preferred the bundle z. Therefore, consumer can choose any bundle in between z and C' under constant real income. If the consumer chooses the bundle z, then we have a single quantity of good 1 with two different prices which is not possible in view of the fact that the demand function is single valued. Hence, under the constant real income consumer actually chooses any bundle on the line A'C' right to the point z, say at point T.

Clearly, x_1 increases from x_1^0 to x_1^1 due to own substitution effect and from x_1^1 to x_1^2 due to income effect for a price change. Therefore, under the theory of revealed preference own price effect is the summation of own substitution effect and income effect for a price change. If we consider Slutsky's method for decomposition of own substitution effect and income effect for a price change in indifference curve approach, we have the similar income effect for a price change. But if we consider Hicksian approach in indifference curve analysis, then income effect for a price change is larger in revealed preference approach for a normal good. This is so because in revealed preference approach we must consider Slutsky's method as there is no utility function and the real income can't be measured in terms of utility.

In the next section, we will mathematically show that own substitution effect is negative.

2.2.4 Determination of the Sign of Substitution Effect

Graphically

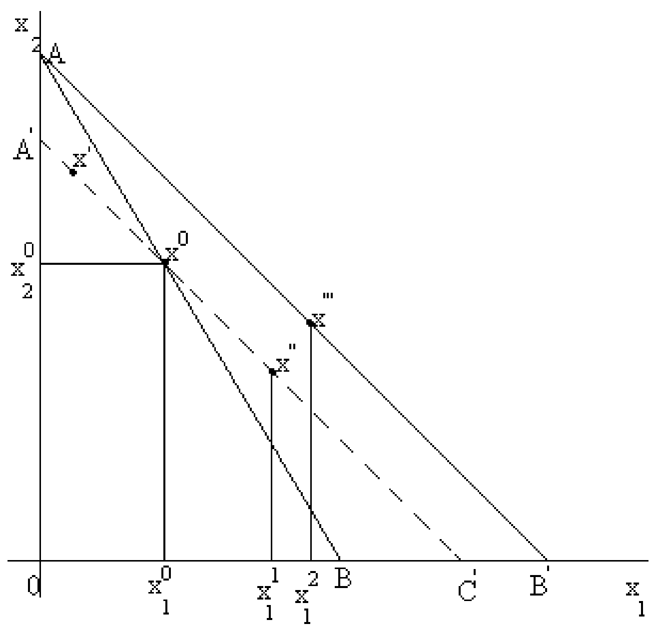

Fig. 2.3: Substitution Effect is Negative

Let the initial price income situation be (p^0, M^0) (where $p^0 = (p_1, p_2)$; p_1 is the price of one unit of good I (x_1) and p_2 is the price of one unit of good II (x_2)] is represented by price line AB in the above figure).

In a two-commodity space in x_1 and x_2, consumer buys bundle x^0. Let p_1 goes down and other things remain unchanged to get the new line AB' which is steeper than the old budget line AB. In the new budget line, equilibrium can be take place anywhere. Let at x"' the equilibrium takes place where $x_1 = x_1^2$. At old equilibrium $x_1 = x_1^0$. Therefore, $(x_1^2 - x_1^0)$ is the change in demand of x_1 due to change p_1, which is nothing but the price effect. The price effect can be partitioned into substitution effect due to change in relative price of x_1, and income effect for a price change due to change in real income. To remove income effect for a price change, we must maintain consumer's real income constant. Since revealed preference theory does not use any concept of utility function, we cannot measure constant real income by indifference curve passing through the initial point of equilibrium x^0. Instead, we fall back upon the Slutsky's concept of constant purchasing power. Initially consumer's purchasing power is $p^0x^0 = M^0$. Constant purchasing power is that amount of money, which enables the consumer to buy the initial bundle at new prices, say p^1 and income M^1. Therefore, $p^1x^0 = M^1$.

To remove the income effect, we draw a hypothetical, compensated, constant purchasing power budget line A'C' in the above figure, passing to the point x^0 and parallel to AB'.

Suppose consumer buys less of x_1, given by bundle x^1, which lies within the old budget space triangle 0AB. In (p^0, M^0) when she bought x^0, x^1 was available, so x^0 was revealed preferred to x^1. But now she is purchasing x^1 when x^0 is available. So x^1 is revealed preferred to x^0. This contradicts the weak axiom of revealed preference. Therefore, new chosen bundle cannot lie to the left of x^0. The consumer must go to right of x^0 to a point like x", if she buys more of x^1. As p_1 goes down, quantity of x^1 purchased increased, so substitution effect is negative. Price change and quantity change move in opposite direction.

Mathematical Presentation

Here we present the mathematical and more general proof of the above result.

Consider, again, the initial price income situation (p^0, M^0), where x^0 is the chosen bundle. Prices change from p^0 to p^1, where $p^0>p^1$, and consumer's real income changes (in this case increases). Suppose we adjust consumer's money income in such a way that her purchasing power remains the same. Consumer's money income is changed to $M^1 = p^1x^0$. Suppose x" is the bundle consumer buys with her adjusted money income viz.,

$M^1 = p^1x''$. Therefore, $M^1 = p^1x^0 = p^1x''$

or, $p^1x^0 - p^1x'' = 0$

or, $p^1 (x^0 - x'') = 0$ ------------------ (a)

Since with (p^1, M^1) consumer chooses x" while x^0 was available, so x" is revealed preferred to x^0 and therefore was not available when x^0 was chosen while (p^0, M^0) prevailed. Thus,

$p^0x'' > p^0x^0$

or, $p^0 (x^0 - x'') < 0$ ------------------- (b)

Adding equation (a) and equation (b) we get,

$p^0 (x'' - x^0) - p^1 (x'' - x^0) > 0$

or, $(p^0 - p^1)(x'' - x^0) > 0$

or, $(p^1 - p^0)(x'' - x^0) < 0$

Suppose only price of i^{th} good changes, other prices remaining constant. Therefore,

$$(p_i^0 - p_i^1)(x_i'' - x_i^0) < 0$$

So, quantity change is in opposite direction of the price change. Hence, substitution effect is negative. Now, $dp = (dp_1, dp_2, \ldots, dp_i, \ldots, dp_n)$ and $dx = (dx_1, dx_2, \ldots, dx_i, \ldots, dx_n)^T$, so that

$$dp\,dx = dp_1 dx_1 + dp_2 dx_2 + \ldots + dp_i dx_i + \ldots + dp_n dx_n$$

Since $dp_1, dp_2, \ldots, dp_{i-1}, dp_{i+1}, \ldots, dp_n$ are all equal to zero, we have

$$dp\,dx = dp_i dx_i < 0$$

So own price substitution effect in negative.

2.2.5 Demand Function is Homogeneous of Degree Zero

Mathematical Presentation

In this section we will show that demand function is homogeneous of degree zero in prices and money income. In other words, if prices and money income change proportionately, then demand for all goods (and therefore the equilibrium bundle) remain the same.

Initial price set $p^0 = (p_1^0, p_2^0, \ldots, p_n^0)$ and the money income is I^0 at which commodity vector x^0 is bought, where $x^0 = (x_1^0, x_2^0, \ldots, x_n^0)$ and $p^0 x^0 = I^0$. Let there be another commodity set $x^1 = (x_1^1, x_2^1, \ldots, x_n^1)$.

At prices p^0 set x^1 is not chosen because it is not more expensive than x^0 and at that price x^1 is available so x^0 is revealed preferred to x^1. So,

$$p^0 x^0 \geq p^0 x^1$$

$$\text{or, } \sum_{i=1}^{n} p_i^0 x_i^0 > \sum_{i=1}^{n} p_i^0 x_i^1$$

$$\text{or, } \sum_{i=1}^{n} p_i^0 (x_i^1 - x_i^0) < 0 \text{ ------------------- (a)}$$

Consider a change in prices to $p^1 = (p_1^1, \ldots, p_n^1)$ and income changes to I^1, when consumer buys x^1. So x^0 at that new price level is not available to the consumer and we have,

$$\sum_{i=1}^{n} p_i^1 x_i^1 < \sum_{i=1}^{n} p_i^1 x_i^0$$

$$\text{or, } \sum_{i=1}^{n} p_i^1 (x_i^1 - x_i^0) < 0 \text{ ------------------- (b)}$$

By weak axiom of revealed preference, when x^1 and x^0 are distinct.

Let $I^1 = m\ I^0$ and $p^1 = m\ p^0$, where m is any positive number. Prices and income change in same proportion m and we have,

$$\sum_{i=1}^{n} p_i^1 x_i^1 = m \sum_{i=1}^{n} p_i^0 x_i^0$$

$$\text{or, } m \sum_{i=1}^{n} p_i^0 x_i^1 = m \sum_{i=1}^{n} p_i^0 x_i^0$$

$$\text{or, } \sum_{i=1}^{n} p_i^0 (x_i^1 - x_i^0) = 0 \text{ ----------------------- (c)}$$

Again,

$$\sum_{i=1}^{n} p_i^1 x_i^1 = m \sum_{i=1}^{n} p_i^0 x_i^0$$

$$\text{or, } m \sum_{i=1}^{n} p_i^0 x_i^0 = \sum_{i=1}^{n} p_i^1 x_i^0$$

$$\text{or, } \sum_{i=1}^{n} p_i^1 (x_i^1 - x_i^0) = 0 \text{ -------------------------- (d)}$$

Equations (c) and (d) contradict equations (a) and (b). Hence, we conclude that the x^1 and x^0 sets are identical and demand functions (which are functions of prices and quantities) are homogeneous of degree zero and same proportional change in prices and income leads an individual to buy the same bundle.

Check Your Progress 1

1) Write down the assumptions of the theory of revealed preference.

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2) Derive the ordinary demand curve. What is the sign of that curve?

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3) Show that just like indifference curve approach, the sign of the substitution effect is negative in case of revealed preference.

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4) What do you mean by a function to be homogeneous of degree zero? Show that ordinary demand function is homogeneous of degree zero.

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2.3 CONSUMER CHOICE INVOLVING RISK

Up to the previous section we have dealt with the theory of consumer behaviour under certainty. The consumer has perfect knowledge for everything regarding market prices, quality of goods and number of commodity that are available to her. In the following we will discuss the consumer behaviour under uncertainty. In the real world there are only few things in market that consumer know perfectly. Therefore, consumer behaviour under uncertainty is a more realistic concept to explain her decision making process than the theory that deals with certainty.

2.3.1 Introduction

The traditional theory of consumer behaviour does not include an analysis of uncertain situation. Von Neumann and Morgenstern showed that under some circumstances it is possible to construct a set of numbers for a particular consumer that can be used to predict her choices in uncertain situations. However, there is a great controversy that has centered around the question of whether the resulting utility index is ordinal or, cardinal. It will be shown that Von Neumann – Morgenstern utilities possess at least some cardinal properties.

It has been pointed out above that consumer behaviour analysis is unrealistic in the sense that it assumes actions the consumer are followed by determinate consequences which are knowable in advance. For instance, all automobiles of the same model and produced in the same factory will not always have the same performance characteristics. As a result of random accidents in the production process, some substandard automobiles could be occasionally produced and sold. The consumer has no way of knowing ahead of time whether the particular automobile, which she purchased, is of standard quality or not. Let A represent the situation in which the consumer possesses a standard quality automobile and B be a situation in which she does not. Again, let there be C, in which she possesses a substandard automobile. Assume that the consumer prefers A to B and B to C. That is, not having a car is assumed preferable to owning a substandard one because of the nuisance and expense involved in its uptake. Present her with a choice between two alternatives: (1) She can maintain the status quo and have no car at all. This is a choice with certain outcome i.e., the probability of the outcome equals unity. (2) She can obtain a lottery ticket with a chance of winning either a satisfactory automobile (alternative A) or an unsatisfactory one (alternative C). The consumer may prefer to retain her income (or money) with certainty, or she may prefer the lottery ticket with dubious outcome, or she may be indifferent between them. Her decision will depend upon the chances of winning or losing in this particular lottery. If the probability of C is very high, she might prefer to retain her money with certainty; if the probability of A is very high, she might prefer the lottery ticket. The triplet of numbers (P, A, B) is used to denote a lottery offering outcome A with probability $0<P<1$, and outcome B with probability 1-P.

2.3.2 Axioms

It is possible to construct a utility index which can be used to predict choice in uncertain situations if the consumer conforms to the following five axioms:

- **Axiom of Complete-ordering:** For the two alternatives A and B one of the following must be true: the consumer prefers A to B, she prefers B to A, or she is indifferent between them. The consumer's evaluation of alternatives is transitive: if she prefers A to B and B to C, she prefers A to C.
- **Axiom of Continuity:** Assume that A is preferred to B and B to C. The axiom asserts that there exists some probability P, $0<P<1$, such that the consumer is indifferent between outcome B with certainty and a lottery ticket (P, A, C).
- **Axiom of Independence:** Assume that the consumer is indifferent between A and B and that C is any outcome whatever. If one lottery ticket L_1 offers outcome A and C with probability P and 1-P respectively and another L_2 the outcomes B and C with the same probabilities P and 1-P, the consumer is indifferent between the two lottery tickets. Similarly, if she prefers A to B, she will prefer L_1 to L_2.
- **Axiom of Unequal-probability:** Assume that the consumer prefers A to B. Let $L_1 = (P_1, A, B)$ and $L_2 = (P_2, A, B)$. The consumer will prefer L_2 to L_1 if and only if $P_2>P_1$.
- **Axiom of Compound-lottery:** Let $L_1 = (P_1, A, B)$, and $L_2 = (P_2, L_3, L_4)$, where $L_3 = (P_3, A, B)$ and $L_4 = (P_4, A, B)$, be a compound lottery in which the prizes are lottery tickets. L_2 is equivalent to L_1 if $P_1 = P_2P_3 + (1-P_2) P_4$. Given L_2 the probability of obtaining L_3 is P_2. Consequently, the probability of obtaining A through L_2 is P_2P_3. Similarly, the probability of obtaining L_4 is $(1-P_2)$, and the probability of obtaining A through L_4 is $(1-P_2)\ P_4$. The probability of obtaining A with L_2 is the sum of the two probabilities. The consumer evaluates lottery tickets only in terms of the

probabilities of obtaining the prizes, and not in terms of how many times she is exposed to a chance mechanism.

These axioms are very general in nature, and it may be difficult to object to them on the grounds that they place unreasonable restrictions upon the consumer's behaviour. However, they rule out some types of plausible behaviour. Consider a person who derives satisfaction from the share of gambling. It is conceivable that there exists no P other than P=1 or P=0 for such a person, so that she is indifferent between outcome B with certainty and the uncertain prospect consisting of A and C; she will always prefer the "sure thing" to the dubious prospect. This type of behaviour is ruled out by the continuity axiom and the compound lottery axiom.

The axioms have been developed for situations in which there are only two outcomes. Assuming that the pair-wise axioms hold, analysis is easily extended to cases with any number of outcomes. Let $L = (P_1,\ldots,P_n, A_1,\ldots,A_n)$ denote a lottery with n outcomes where $0<P_i<1$ is the probability of outcome A_i and $\sum_{i=1}^{n} Pi = 1$.

2.3.3 Expected Utility Theory

Assume that a utility index exists which conforms to the five axioms. The expected utility for the two-outcome lottery L = (P, A, B) is given by,

$$E[U(L)] = P\,U(A) + (1-P)\,U(B)$$

Consider the lotteries $L_1 = (P_1, A_1, A_2)$ and $L_2 = (P_2, A_3, A_4)$. An expected utility theorem states that if L_1 is preferred to L_2, $E[U(L_1)] > E[U(L_2)]$. The significance of this theorem is that uncertain situations can be analysed in terms of the maximisation of expected utility.

The proof of the theorem is straightforward. Select outcomes such that B, the best, is preferred to all other outcomes under consideration, and W, the worst, is inferior to all other outcomes. By the continuity axiom, Q_i's exist such that A_i is indifferent to (Q_i, B, W) (i=1,…,4). Thus L_1 and L_2 are equivalent to, i.e., have the same expected utility as, the lotteries (Z_1, B, W) and (Z_2, B, W) respectively where $Z_1 = P_1Q_1 + (1-P_1)Q_2$ and $Z_2 = P_2Q_3 + (1-P_2)Q_4$. By assumption, L_1 is preferred to L_2 and it follows from the unequal probability axiom that $Z_1>Z_2$. Since origin and unit of measure are arbitrary for utility indexes, let U (B) = 1 and U (W) = 0. Then $E[U(L_1)] = Z_1$ and $E[U(L_2)] = Z_2$, establish the theorem.

2.3.4 Attitude towards Risk

Let's assume the following:

The utility function

- has the single argument "wealth" measured in monetary units,
- is strictly increasing, and
- is continuous with continuous first order and second order derivatives.

The expected value of the lottery (P, W_1, W_2), where the Wi are the different wealth levels, is the sum of the outcomes, each multiplied by its probability of occurrence. Thus,

$$E[W] = PW_1 + (1-P)W_2$$

A person is risk neutral relative to a lottery if its utility of the expected value equals the expected utility of the lottery, i.e., if

$$U[PW_1 + (1-P)W_2] = PU(W_1) + (1-P)U(W_2) \text{ ---------------- (a)}$$

Such a person is only interested in expected values and is totally oblivious to risk. If she is risk neutral towards all lotteries, equation (a) implies that she has a linear utility function of the form $U = \alpha + \beta W$ with $\beta>0$. The utility analysis developed for certain situations is applicable for risk-neutral persons facing uncertainty. All that is necessary is to replace certain values with expected values.

A person is a risk averter relative to a lottery if the utility of its expected value is greater than the expected value of its utility:

$$U[PW_1 + (1-P)W_2] > PU(W_1) + (1-P)U(W_2) \text{ -------------- (b)}$$

Such a person prefers a certain outcome to an uncertain one with the same expected value. If equation (b) holds for all $0<P<1$ and all W_1 and W_2 within the domain of the utility function, the utility function is strictly concave over its domain since equation (b) is identical to the definition of strict concavity.

2.3.5 Summary

The approach of Von Neumann and Morgenstern is concerned with the consumer's behaviour in situations characterised by uncertainty. If the consumer's behaviour satisfies some crucial axioms, her utility function can be derived by presenting her with a series of choices between a certain outcome on the one hand and a probabilistic combination of two uncertain outcomes on the other hand. The utility function thus derived is unique up to a linear transformation, and provides a ranking of alternatives in situations that do not involve risk. Consumers maximise expected utility, and Von Neumann-Morgenstern utilities are cardinal in the sense that they can be combined to calculate expected utilities and can be used to compare differences in utilities. The expected utility calculation can be used to determine the consumer's choices in situations involving risk.

With wealth as the single argument of the utility function, the utility of the expected value of the outcome of an uncertain situation exceeds the expected utility of the outcome for a risk-averse consumer; i.e., her utility function is strictly concave. Similarly, risk lovers and risk neutrals have strictly convex and linear utility functions respectively.

Check Your Progress 2

1) Write down the assumptions of the theory of consumer behaviour under risk.

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2) Define risk neutral, risk averse and risk loving consumer.

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2.4 INDIRECT UTILITY FUNCTION AND DUALITY THEORY

In this section we go back to the world of certainty. There are some interesting features of the theory of consumer behaviour under certainty and our objective here is to discuss some of them in details. First, we will discuss indirect utility function before the duality theorems.

2.4.1 Indirect Utility Functions

Let q_i denotes commodity i and p_i is the price of that commodity. Let y denotes money income of the consumer. Suppose $v_i = p_i/y$. The budget constraint now may be written as

$$1 = \sum_{i=1}^{n} v_i q_i \text{ -------------------- (a)}$$

Since optimal solutions in the demand functions are homogeneous of degree zero in income and prices, nothing essential is lost by this transformation to “normalised” prices. The utility function $U = f(q_1, q_n)$ together with equation (a) gives the following first order conditions of utility maximisation:

$$f_i - \lambda v_i = 0 \text{ for all } i = 1,\ldots, n$$

$$\text{and } 1 = \sum_{i=1}^{n} v_i q_i \text{ ------------------ (b)}$$

Ordinary demand functions are obtained by solving equation (b):

$$q_i = D_i (v_1,\ldots,v_n)$$

The indirect utility function $g(v_1,\ldots, v_n)$ is defined by

$$U = f[D_1(v_1,\ldots,v_n),\ldots, D_n(v_1,\ldots,v_n)] = g(v_1,\ldots,v_n) \text{ ------------------ (c)}$$

It gives the maximum utility as a function of normalised prices. The direct utility function describes preferences independent of market phenomena. The indirect utility function reflects a degree of optimisation and market prices.

Applying the composite function rule of calculus to equation (c), we get

$$g_j = \sum_{i=1}^{n} f_i \frac{\partial q_i}{\partial v_i} = \lambda \sum_{i=1}^{n} v_i \frac{\partial q_i}{\partial v_j} \quad j = 1, \ldots, n$$ ---------------------------(d)

where the second equalities are based on equation (b). Partial differentiation of equation (a) with respect to v_j yields

$$\sum_{i=1}^{n} v_i \frac{\partial q_i}{\partial v_j} = -q_j \quad j = 1, \ldots, n$$

Thus, equation (d) implies

$q_j = -(g_i/\lambda)$ j = 1,…,n ------------------------ (e)

which is called the Roy's identity. Optimal commodity demands are related to the derivatives of the indirect utility function and the optimal value of the Lagrange multiplier (i.e., the marginal utility of income). Substituting equation (e) into the last equation of equation (b) gives

$$\lambda = -\sum_{i=1}^{n} v_i g_i$$

and $q_j = \dfrac{g_i}{\sum_{i=1}^{n} v_i g_i}$ j = 1,…,n

to provide an alternative form of Roy's identity.

Now consider an optimisation problem in which equation (c) is minimised subject to equation (a) with normalised prices as variables and quantities as parameters. From the function, $Z = g(v_1, \ldots, v_n) + \mu(\sum_{i=1}^{n} v_i q_i - 1)$

and setting its partials equal to zero, we get

$$\frac{\partial Z}{\partial v_i} = g_i - \mu q_i \quad i = 1, \ldots, n$$

and

$$\frac{\partial Z}{\partial v_i} = \sum_{i=1}^{n} v_i q_i - 1 = 0$$ --------------------------------- (f)

"Inverse demand functions" are obtained by solving equation (f) for the prices as functions of quantities:

$v_i = V_i(q_1, \ldots, q_n)$

Finally, let a direct utility function h $(q_1, \ldots, q_n)$ be defined by

$U = g\,[V_1(q_1, \ldots, q_n), \ldots, V_n(q_1, \ldots, q_n)] = h(q_1, \ldots, q_n)$ – (g)

This provides a parallel to the direct problem in which quantities are variables and prices are parameters.

2.4.2 Duality Theorems

The relationship between the direct and indirect utility functions may be described by a set of duality theorems. The following illustrative theorems are provided without proof.

Theorem 1: Let f be the finite regular strictly quasi-concave increasing function which obeys the interior assumption (the utility for a commodity combination in which one or more quantities is zero is lower than the utility for any combination in which all quantities are positive). The g determined by equation (c) is a finite regular strictly quassi-convex decreasing function for positive prices.

Theorem 2: Let g be a finite regular strictly quassi-convex decreasing function in positive prices. The h determined by equation (g) is a finite regular strictly quassi-concave increasing function.

Theorem 3: Under the above assumptions

$h(q_1,\ldots,q_n) = g[V_1(q_1,\ldots,q_n),\ldots, V_n(q_1,\ldots,q_n)]$

and $g(q_1,\ldots, q_n) = h[D_1(q_1,\ldots, q_n),\ldots, D_1(q_1,\ldots, q_n)]$

The direct utility function determined by the indirect is the same as the direct utility function that determined the indirect.

Duality in consumption forges a much closer link between demand and utility functions for the purposes of empirical demand studies. It is sometimes possible to go from demand functions to the indirect utility function by using Roy's identity, and then to the corresponding direct utility function. Duality is also useful in comparative statics analysis. Homotheticity, separability, and additivity each have counterparts for the indirect utility function. Consequently, many theoretical analyses can be conducted in terms of either the direct or indirect utility function, whichever is more convenient.

2.4.3 Utility-Expenditure Duality

Consider the minimisation of the expenditures necessary to achieve a specified utility level. The solution for q_i yields the compensated demand functions. If the solutions for q_i are substituted in $\sum_{i=1}^{n} p_i q_i$ one obtains the expenditure function E $(p_1,\ldots,p_n,U^0)$, which gives the minimum expenditure necessary to achieve a given utility level. It is easy to show that E is homogeneous of degree one in prices and monotonically increasing in U^0. It can also be shown that the expenditure function corresponding to a regular strictly quassi-concave utility function admitting no satiation is concave in prices. Finally, Shephard's lemma states that the partial derivative of E with respect to the i^{th} price is the i^{th} compensated demand function. This can be shown as follows:

Denote the i^{th} compensated demand function by

$q_i = q_i\,(p_1,\ldots,p_n,U^0)$.

Then

$$E\,(p_1,p_n,U^0) = \sum_{i=1}^{n} p_i q_i (p_1,\ldots,p_n,U^0)$$

and

$$\frac{\partial E}{\partial p_i} = q_i\left(p_1,\ \ldots,\ p_n,\ U^0\right) + \sum_{j=1}^{n} p_j\ \frac{\partial q_j\left(p_1,\ \ldots,\ p_j,\ U^0\right)}{\partial p_i}$$

But the compensated demands are obtained by minimising expenditures for a given utility level U^0; hence the change in total expenditures that is due to a small change in a price is zero. It follows that the second term above is zero and

$$\frac{\partial E}{\partial p_i} = q_i\left(p_1,\ \ldots,\ p_n,\ U^0\right)$$

The duality between utility and expenditure functions is formally identical to the duality between production and cost functions.

Check Your Progress 3

1) Write down the duality theorems.

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2) Consider an indirect utility function $g = a - v_1^2 v_2$. Derive the demand functions for good I and good II and thus derive the direct utility function.

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2.5 LET US SUM UP

The most famous approach in the history of consumer behaviour, after indifference curve approach, is the revealed preference approach. In the revealed preference approach there is no concept of utility function and that is why the approach has no concept of indifference curve. The consumer has only preferences i.e., between any two commodities A and B, she can either prefer A over B or prefer B over A or A and B give same level of utility to the consumer, i.e. consumer is indifference between A and B. The slope of the demand curve of a good can take any algebrical sign. There are five main axioms of revealed preference theory. In revealed preference approach own price effect can be decomposed into own substitution effect and income effect for a price change. Substitution effect is negative. The demand function is homogeneous of degree zero with respect to prices and money income, i.e., if prices of all goods and money income change proportionately then demand for each commodity remains unchanged, some times this can be put forward as ‘Consumer is free from money illusion’.

In uncertain world consumer’s objective is to maximise expected utility unlike the certain world where her objective is to maximise her utility. The consumer preferences can be completely described by five axioms.

There is a duality between utility maximisation and expenditure minimisation, i.e., they both gives the same results. There are three duality theorems. Roy’s identity relates optimal commodity demands to the derivatives of the indirect utility function and the optimal value of the Lagrange multiplier.

2.6 SOME USEFUL BOOKS

Demand Function - Homogenous of Degree Zero: Prices and income changing proportionately to leave quantity demanded unchanged.

Duality: The relationship between minimisation and maximisation objectives.

Expected Utility: An average utility expected from an uncertain situation.

Homogenous Function: A function, $f(x_1, x_2, \ldots\ldots, x_n)$ is homogenous of degree k, if $f(mx_1, mx_2, \ldots\ldots, mx_n) = m^k f(x_1, x_2, \ldots\ldots, x_n)$.

Indirect Utilisation Function: A formulation in which utility is postulated as a function of all prices and income.

2.7 SOME USEFUL BOOKS

J., Henderson & Richard E. Quandt (2003), *Microeconomic Theory: Mathematical Approach*, Tata McGraw-Hill Publishing Company Limited, New Delhi.

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillian.

Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York.

2.8 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) See sub-section 2.2.2.

2) See sub-section 2.2.3. The demand curves can any algebrical sign.

3) See sub-section 2.2.4.

4) See sub-section 2.2.5.

Check Your Progress 2

1) See sub-section 2.3.2.

2) See sub-section 2.3.5.

Check Your Progress 3

1) See sub-section 2.4.2

2) Note that the indirect utility function is quassi-concave and the indifference curves are downward sloping and strictly convex to the origin.

   Demand curves for the exercise can be derived from equation (e), which is nothing but the famous Roy's identity,

   $$q_1 = \frac{2}{3v_1} \quad q_2 = \frac{1}{3v_2}$$

   therefore, we have ,

   $$v_1 = \frac{2}{3p_1} \qquad v_2 = \frac{1}{3p_2}$$

   and that the corresponding direct utility function is

   $$U = a - \left(\frac{2}{3q_1}\right)^2 \frac{1}{3q_2} = a - \frac{4}{27q_1{}^2 q_2}$$

   which is a strictly quassi-concave increasing function.