UNIT 1 THEORY OF COSUMER BEHAVIOUR: BASIC THEMES

Structure

1.0 OBJECTIVES

The objective of this unit is to relate how individual consumers take decisions of consumption in a situation where market prices are given to them and they can't influence the market prices by altering their consumption. This unit will enable you to:

- Determine the optimum choice of a consumer;
- Explain how the price effect can be decompose into income effect and substitution effect; and
- Determine the individual demand curve.

1.1 INTRODUCTION

It is generally observed that market aggregate demand curve for a commodity is downward sloping, given other things. Our problem is to investigate economic rationality behind this for a commodity of all individual consumers. The market demand basically depends on the characteristics of demand for a commodity by individual consumers, and the demand for a commodity of an individual consumer depends upon the behaviour of the consumer. Clearly, to investigate economic rationality behind the law of demand, we shall start with the analysis of consumer behaviour.

1.2 THE BASIC THEMES

There are different approaches to analyse the consumer behaviour. But in all approaches, it is assumed that the consumer is rational. This means that the consumer's objective is to maximise her utility by choosing one commodity bundle from among all the commodity bundles (money income and the prices of the commodities are given to the consumer).

1.3 CONSUMER CHOICE CONCERNING UTILITY

Consumers can't maximise her utility unless she can measure it. Hence, utility must be a measurable concept. The measurement is undertaken differently in different approaches. In traditional frame, we have two types of measurement of utility,

1) Cardinal analysis
2) Ordinal analysis

1.3.1 Cardinal Theory: An Introduction

In cardinal approach, utility is measured cardinally or numerically in terms of money. The consumer not only knows which one is preferred but also by what amount. The assumptions of this approach is given below:

1) Consumer is rational.

Implication: The consumer's objective is to maximise her utility by choosing one of the commodity bundle from all other available commodity bundles at given prices of commodities and money income.

2) If the taste and preferences are given, the total utility of the consumer depends on the quantity of consumption.
3) Goods are good.

Implication: Let ‘U’ denote utility level of the consumer and let ‘x’ be the consumption bundle. As ‘x’ increases (decreases) ‘U’ increases (decreases). Therefore, marginal utility is positive.

4) Marginal utility of ‘x’ is diminishing.

Implication: As ‘x’ increases (decreases) MU_x decreases (increases). Therefore, MU_x curve is downward sloping

5) Utility is measured cardinally or numerically in terms of money.

Implication: Since it is measured numerically consumer not only knows which commodity bundle is preferred but also by how much amount.

6) Marginal utility of money is constant.

Implication: $MU_m = \lambda$ where λ is positive and constant. That means as money income increases (decreases) by one unit, utility increases (decreases) by λ unit.

Consumer Equilibrium:

According to our assumption for 'x' units consumption of the commodity, gross utility obtained by the consumer is U(x).But for this, the consumer must spend $p_x.x$ units of money income if p_x be the price of the commodity 'x', which is given to the consumer. Since from assumption 6, λ represents fall in utility due to one unit fall in money income, the net utility of the consumer is given by $N(x) = U(x)-\tilde{\lambda}p_x.x$, where λ and p_x are given to the consumer. So consumer's objective is to maximise N(x) by choosing 'x'. For that we take the first derivative of N(x) and set that equal to zero, $\frac{dN(x)}{dx} = 0$.Or, we get $\frac{dU(x)}{dx} - \lambda p_x = 0$. From this first order condition, we can derive the optimum value of 'x' which is (say) $x^* = x^*(p_x,\lambda)$. The second order condition for utility Maximisation requires $\frac{\partial^2 N(x)}{\partial x^2} = \frac{\partial^2 U(x)}{\partial x^2} < 0$, which is ensured by the assumption of falling MU_x.

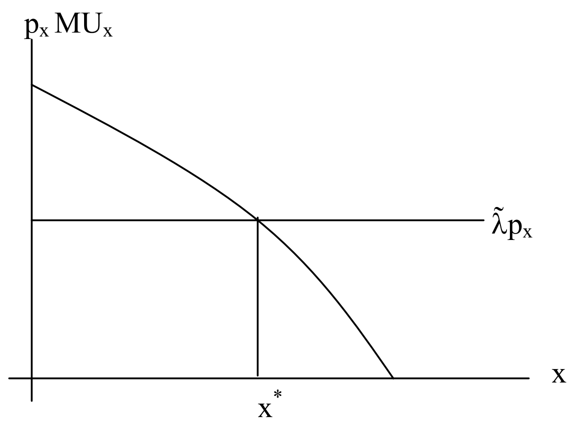

Fig. 1.1: Consumer Equilibrium in Cardinal Theory

Check Your Progress 1

1) What are the assumptions of cardinal utility theory?

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2) Consider the utility function U (x) = log (x), let $p_x = 2$ and $\lambda = 5$. Derive the consumer equilibrium and check the second order condition.

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1.3.2 Codinal Theory: A Short Note

In ordinal approach, utility is measured ordinally i.e., qualitatively (not numerically or quantitatively). Alternatively, consumer can rank her preferences according to the order she wants to compare but not in terms of the different amount. It's a qualitative measure and therefore more realistic measurement of utility or satisfaction.

There are two different approaches of ordinal theory, viz.,

1) Indifference curve approach
2) Revealed preference approach

1.3.2.1 Indifference Curve Approach

Indifference curve is constructed by taking utility level constant, so different indifference curves imply different level of utility for same consumer. The equilibrium is achieved when indifference curve become tangent to the budget line.

1.3.2.2 Revealed Preference Approach

In revealed preference approach, consumer equilibrium can be found by ranking different bundle of goods in the commodity space. Given the budget constraint, consumer chooses the best bundle for which her utility will maximise. This theory was originally constructed by the famous economist Paul. A. Samuelson.

1.4 INTRODUCTION TO DEMAND ANALYSIS

It is generally seen that market demand curve is downward sloping. Market demand curve (or sometimes called Aggregate demand curve) is nothing but the aggregation of individual demand curves. Individual demand curve can be constructed by joining different consumer equilibrium for different prices (remember that consumer can't alter the market prices, it is given to the consumer). In neo-classical consumer theory, price is exogenous variable, so demand curve can be obtain only if we change the price exogenously and join all the equilibrium points. From next on our objective is to find out the consumer demand curve, for which we will adopt ordinal theory and in that, we will take indifference curve approach.

1.5 ORDINAL THEORY: INDIFFERENCE CURVE APPROACH

In indifference curve approach consumer is assumed to be rational, so that consumer's objective is to maximise her utility by choosing a commodity bundle among all other available commodity bundles (under budget constraint) where total utility ('U') depends on quantity consumption given her taste and preferences. Therefore, in a two-commodity world (say x_1 and

x_2) utility function is given by U = U (x_1,x_2) and it depends on taste and preferences of the consumer, which is specified by axioms given below:

1) **Axiom of reflexiveness:** Consumer's choice is reflexive.

Implication: Weak preference relation is denoted by 'R'. Suppose there are two goods x_1 and x_2 and suppose x_1 is weakly preferred to x_2 i.e., x_1Rx_2 which implies that either x_1 is strictly preferred over x_2 (it is denoted by x_1Px_2) or x_1 is indifference to x_2 (it is denoted by x_1Ix_2), where 'P' and 'I' implies strict preference relation and indifference respectively.

The set constituted by all commodity bundles or vector is known as commodity set (X). Any one commodity bundle is denoted by 'x' is weakly preferred (i.e., either strictly preferred or indifferent) over any other commodity bundle (i.e., in respect to 'x'). Therefore, we have xRx.

Clearly, any one commodity bundle may be indifferent to another commodity bundle i.e., there is a possibility of indifference or same level of utility between the commodity bundles.

None of the commodity bundles are not preferred i.e., consumer can choose any commodity bundle. So choice set of this consumer is specified by the commodity set 'X'.

2) **Axiom of completeness:** Consumer's choice is complete.

Implication: Since consumer is rational, she must have a unique preference relation. That means the consumer choice is either x_1Rx_2 or x_2Rx_1. Alternatively, consumer's choice is consistent or comparable. For unique preference relation, consumer choice must be transitive, where transitivity implies that if x_1Rx_2 and x_2Rx_3 then x_1Rx_3, where x_3 is another commodity.

3) **Axiom of continuity:** Consumer's preference relation (R) is continuous.

1) **Axiom of non-satiation:** Consumer's choice is non-satiated in all goods.

Implication: Non-satiation means larger the consumption of a good leads to larger satisfaction or utility or lower the consumption lower is the satisfaction or utility. Non-satiation of all goods (which means "goods are good" or "more is better") means any commodity bundle 'A' is preferred over another commodity bundle 'B' only if bundle 'A' consists larger quantity of at least one good and no less quantity of any other goods. Notationaly if A>B, then A is preferred over B or APB where B is any other commodity bundle.

2) **Axiom of convexity:** Consumer choice is such that indifference curve is strictly convex to the origin (i.e., utility function is quassi-concave).

3) **Axiom of selfishness:** Consumer choice is selfish.

Implication: Consumer's choice is self-guided. It is not influenced by any other consumer.

1.5.1 Concept of Preference, Utility Function and Indifference Curve

Consumer preference ('R') specified by the above axioms can be represented by a function where total utility ('U') depends on quantity consumption (x_1, x_2), which satisfied all other axioms. The function U = U(x_1, x2) is known as utility function. Since consumer is rational, her objective is to maximise the utility specified by the utility function $U = U(x_1, x2)$ subject to her budget constraint. To solve the consumer utility Maximisation problem, we use a graphical tool, which is known as Indifference curve.

Meaning and definition of indifference curve: Different combination of goods x_1 and x_2 along which consumer is indifferent (or consumer has same level of utility) give a curve in commodity-commodity plane known as indifference curve. Therefore, along the indifference curve utility or satisfaction remains unchanged.

Existence of indifference curve: Because of axiom of reflexiveness consumer can choose a commodity bundle over another commodity bundle i.e., consumer may be indifferent between any commodity bundle and such a choice might be continuous. So, indifference curve may exits anywhere in the commodity space.

1.5.2 Derivation of Indifference Curve

Graphical Presentation

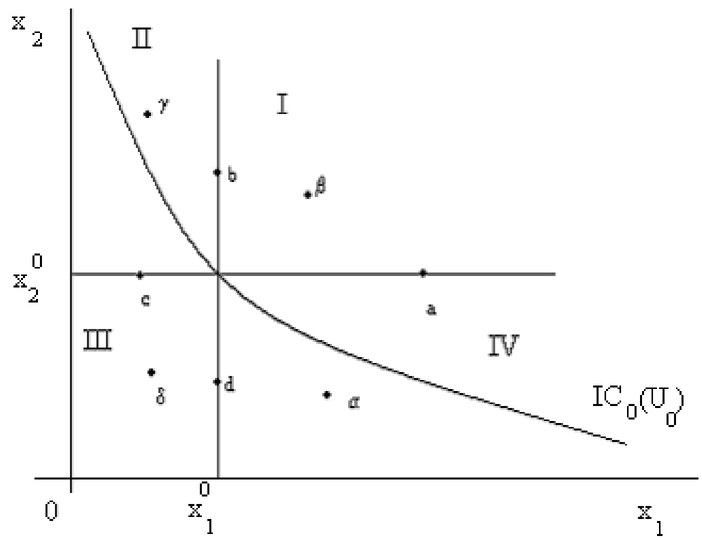

Fig. 1.2: A Typical Indifference Curve

Consider any commodity bundle denoted by point A in the above figure which consist x^0_1

and x^0_2 amount of good I and good II respectively and from which consumer obtains particular level of utility, say U_0. We compare the commodity bundle 'A' with other commodity bundle in the commodity space. For that we divide the entire commodity plane into four phases from 'A'.

Consider any point in phase I say β, where we have large quantity of both x_1 and x_2 compared to point 'A'. Again, if we consider any point say 'a' in horizontal line in phase I, we have larger quantity of x_1 with same quantity of x_2 compared to point 'A'. Similarly, for any point 'b' in vertical axis, we have larger x_2 with same x_1. That means in phase I including the borderlines, we have larger quantity of at least one commodity and no less quantity of any other commodity compared to 'A'. Thus, we have larger utility in phase I including the borderlines compared to 'A'.

By similar logic, we have lower consumption of at least one good and no larger consumption of any other good in phase III including the borderlines compared to point 'A'. Hence, we have lower level of utility in phase III including the borderlines compared to 'A' by the axiom of non-satiation for all goods.

Clearly, in phases I and III, including borderlines, utility is not constant between the commodity bundles compared to point 'A'. So, indifference curve (along which utility is constant) can't pass through phases I and III including their borderlines.

Consider any point in phase IV excluding borderlines, say α. We have larger x_1 (for which utility is larger) and lower x_2 (for which utility is lower) compared to 'A'. Since both goods are non-satiated, utility of point α may be larger, lower or equal compared to point 'A'. Similarly, for any point in phase II excluding the borderlines, say δ, we have larger consumption of x_2 but lower of x_1 compared to point 'A'. Therefore, by axiom of non-satiation in all goods, utility at point δ may be larger, lower or equal compared to 'A'.

Clearly, only in phases II and IV excluding the borderlines, there is a possibility of the same level of utility between the bundles compared to point 'A'. So, indifference curve, along which utility remains unchanged, must pass through the phase II and phase IV, excluding their lines. Thus, indifference curve is necessarily downward sloping where all goods are non-satiated given that a consumer choice is continuous, reflexive and complete.

Mathematical Presentation

Consider the utility function $U = U(x_1, x_2)$. Differentiating totally, we get the following:

$dU = U_1dx_1 + U_2dx_2 = 0$ (as along the indifference curve utility is constant, $dU = 0$). Therefore,

$\frac{dx_2}{dx_1} = -\frac{U_1(x_1, x_2)}{U_2(x_1, x_2)}$, which is the slope of the indifference curve. It is negative since $U_1(x_1, x_2) > 0$ and $U_2(x_1, x_2) > 0$ by assumption of non-satiation of all goods. Thus, indifference curve is downward sloping because all goods are non-satiated, choice is continuous, reflexive and complete.

Economic meaning

All goods are non-satiated i.e., larger (lower) consumption leads to lager (lower) utility. Hence, for given x_2, as x_1 increases, utility increases. Thus, to maintain same level, utility must be reduced, which is possible by reducing x_2. Hence, as x_1 increases, x_2 must decreases in order to maintain same level of utility. That is why indifference curve is downward sloping.

Properties of indifference curve

Property I: Higher indifference curve gives higher utility.

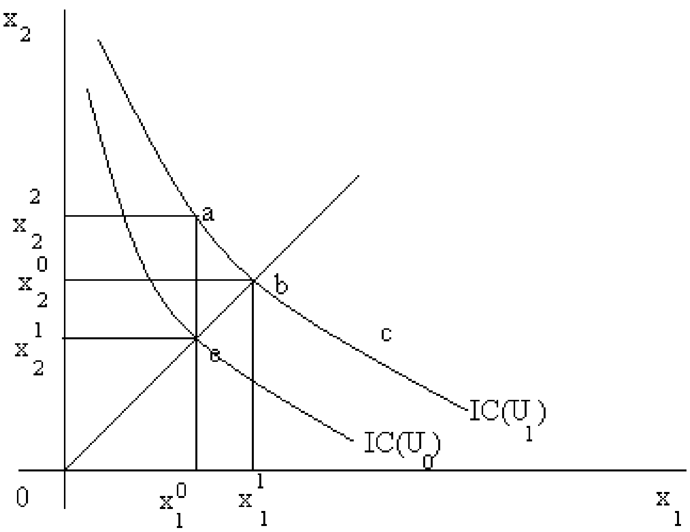

Fig. 1.3: Higher Indifference Curve gives Higher Level of Utility

Explanation: Since all goods are non-satiated, larger consumption of any good leads to larger utility. Thus, a commodity bundle, which consists of larger quantity of at least one good and no less consumption of any other goods, gives larger utility compared to any other commodity bundle. Consequently, higher indifference curve represents higher consumption of at least one commodity and no less consumption of any other commodity.

Property II: Indifference curves can't intersect with each other.

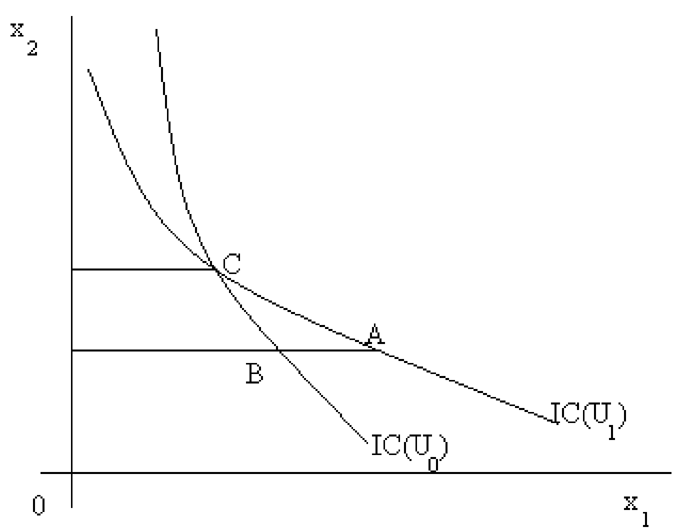

Fig. 1.4: Indifference Curves Can't Intersect Each Other

Explanation: Suppose two indifference curves intersect each other. By definition, along the indifference curve, utility is constant. So, consumer is indifferent between points 'A' and 'C' that lie on the same indifference curve. Similarly, consumer is indifferent between points 'B' and 'C', as they also lie on the same indifference curve. So, AIC and BIC, where 'I' denotes indifference. Now, from transitivity we have AIB i.e., point 'A' and point 'B' give the same utility to the consumer. But for given x_2, x_1 is larger in point 'A' compared to point 'B'. So, by the assumption of non-satiation, we have point

'A' that gives lager utility to consumer as compared to point 'B'. This contradicts the fact that point 'A' and 'B' gives the same level of utility to the consumer (as we have proved above). Therefore, when all goods are non-satiated and transitivity holds, indifference curves can't intersect.

1.5.3 Utility Maximisation

Graphical Presentation

Let consider a two-commodity world, x_1 and x_2 representing good I and good II respectively. p_1 and p_2 are the prices of good I and good II respectively, where the prices are given to the consumer, i.e., prices are exogenously given and consumer can't change them. Money income of the consumer is M, which is also exogenously given to the consumer. Note that $p_1x_1+p_2x_2$ is the total expenditure of the consumer when she consumes $x_1$units of good I and x_2 units of good two. The total expenditure of the consumer can't exceed her money income, therefore $p_1x_1 + p_2x_2 \leq M$ ------- (a)

Equation (a) is known as consumer budget constraint. Let U = U(x_1, x_2) is the utility function of the consumer. Therefore, consumer must solve the following Maximisation problem(UMP):

Problem UMP: Max U(x_1, x_2)

subject to x1 > 0

$x_2 > 0$

and $p_1x_1 + p_2x_2 \leq M$

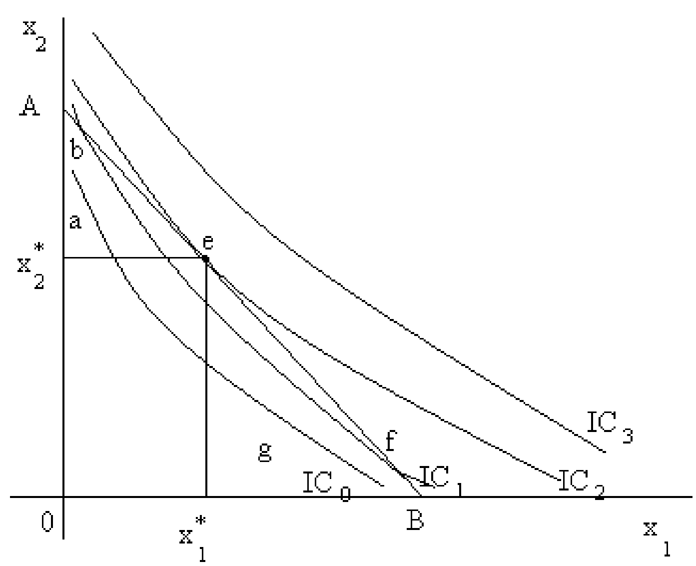

Fig. 1.5: Derivation of Consumer Equilibrium

As consumer objective is to maximise her utility and as larger consumption leads to larger utility, she always wants to consume more of any goods. But she also has to spend some amount of her income to consume larger amount of goods. So ultimately in equilibrium she will spend all her income and M = $p_1x_1+p_2x_2$.

Now suppose that the line segment AB represents the budget line. Along AB $p_1x_1+p_2x_2=M$ holds. Let initial indifference curve of the consumer is IC_0. In IC_0, there are many points along that indifference curve such that $p_1x_1+p_2x_2 \le M$ holds. Therefore, utility maximising consumer will spend more as she moves to higher indifference curve (say IC_1). In IC_1 there are still such points along the indifference curve such that $p_1x_1+p_2x_2 \le M$ holds, so again consumer spends more. This process will continue as long as consumer reaches an indifference curve where for no point along the indifference curve $p_1x_1+p_2x_2 \le M$ holds and at least one point of the indifference curve is on the budget line. At that point, we have consumer equilibrium, $C(x_1, x_2) = (x_1^*(M,p_1,p_2), x_2^*(M,p_1,p_2))$(in Figure 1.5.3 point 'e' is the equilibrium point). Not that at equilibrium, slope of the indifference curve is equal to the slope of the budget line. Therefore, at equilibrium we have

1) Budget constraint holds with equality sign.
2) Slope of the indifference curve is equal to the slope of the budget line.

Mathematical Presentation

Consumer's objective is to maximise her utility by solving UMP. To solve UMP, we set the Lagrange function of the corresponding problem, which is,

$$L(x_1, x_2) = U(x_1, x_2) + \lambda\,(M-p_1x_1-p_2x_2)$$

Our objective is to maximise this Lagrange function by choosing x_1, x_2 and λ. For that we differentiate the Lagrange function by x_1, x_2 and λ, and set all equal to zero.

$$\frac{dL(x_1,x_2)}{dx_1} = \frac{dU(x_1,x_2)}{dx_1} - \lambda p_1 = 0 \text{ ------------- } (f_1)$$

$$\frac{dL(x_1,x_2)}{dx_2} = \frac{dU(x_1,x_2)}{dx_2} - \lambda p_2 = 0 \text{ ------------- } (f_2)$$

$$\frac{dL(x_1,x_2)}{d\lambda} = M - p_1x_1 - p_2x_2 = 0 \text{ ------------- } (f_3)$$

From equation (f1) and (f2), we get,

$\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2} = p_1/p_2$. Note $\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2}$ is the slope of the indifference curve and p_1/p_2 is the slope of the budget line. So, at equilibrium we have a slope of the indifference curve that is equal to the slope of the budget line. Again, from equation (f3) we get $M = p_1x_1+p_2x_2$, so budget equation holds with equality sign.

Check Your Progress 2

1) Define indifference curve in one sentence. What are measured in the axes of the figure to draw an indifference curve?

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2) If $U(x_1, x2) = 10x_1^{0.3}x_2^{0.7}$, M=200, p_1=5 and p_2=2, set up the Lagrange function and derive the simplest form of $\frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} = p_1 / p_2$.

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3) If $U(x_1, x2) = 10x_1^{0.5}x_2^{0.5}$, M=100, p_1=2 and p_2=4 calculate the consumer equilibrium.

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1.5.4 Concepts of Income and Substitution Effects

Change in demand for a good due to one unit change in price of that good for given prices and money income is known as own price effect for that good. Thus, own price effect $= \frac{dx_1}{dp_1}$ and it consists of own substitution effect and income effect for a price change.

Own ***Substitution Effect***: Change in demand quantity for a good (say x_1) due to change in its own price under constant real income (in terms of utility) is called substitution effect for that good and can be written as $(\frac{dx_i}{dp_i})_{\bar{U},\, p_j}$.

Income Effect: Income effect for a good (say x_1) represents change in demand quantity for that good for a change in real income. So income effect = $(\frac{dx_i}{dM})_{\bar{p}}$, which is positive for a normal good, negative for inferior good and zero for neutral good.

Income Effect For A Price Change: For given money income, as price of any one good change one unit then real income (M/p_i) changes for which demand for the good changes by income effect. It is known as income effect for a price change. Thus, income effect for a price change $= -x_i(\frac{dx_i}{dM})$. Note that income effect and income effect for a price change have opposite sign and different magnitude.

1.5.5 Slutsky's Theorem

Graphical Presentation

We prove here that own price effect is the sum of own substitution effect and income effect for a price change, which is known as Slutsky's theorem. This is shown in the figure given bellow:

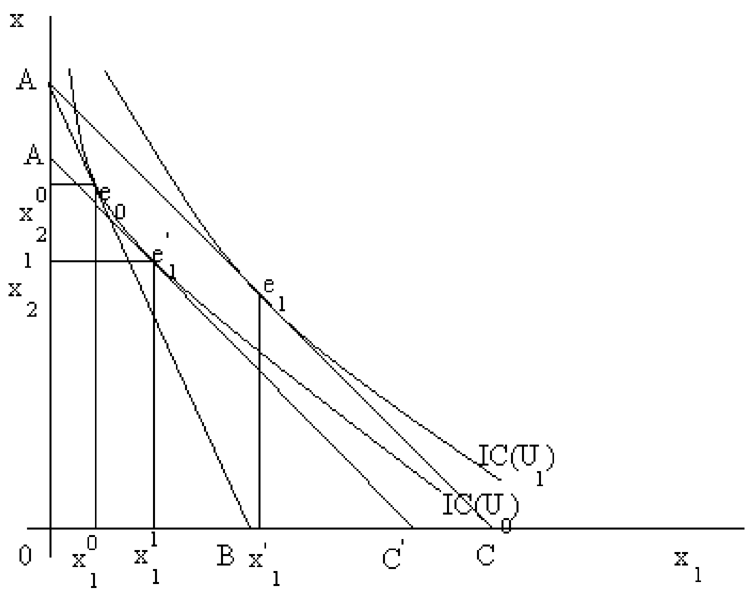

Fig. 1.6: Slustky's Theorem

At initial prices and money income, budget line is AB and according to the condition of the equilibrium e_0 is the initial equilibrium point. The consumer gets U_0 level of utility. Suppose at constant income and p_2, p_1 decreases (say by one unit). Consequently, the intercept of the budget line (M/p_2) remains unchanged but absolute slope of the budget line (p_1/p_2) decreases. The new budget line becomes flatter with the same intercept. It is denoted by AC line. New equilibrium can be achieved at any point on the new budget line AC (and therefore own price effect can take any algebraic sign). Suppose the equilibrium takes place at point e_1. Hence, as p_1 decreases, for given p_2 and M, demand for good I increases from x_1^0 to x_1^1. This is the own price effect for x_1 and here it is negative. A part of this change is due to change in real income (since for given p_2 and M as p_1 decreases, real income increases) and another part is originated at constant real income. To decompose these effects, we reduce money income (M) of the consumer in such a way that real income in terms of utility remains unchanged. After such reduction of M, intercept of the new budget line AC, i.e., (M/p_2) decreases with the same slope (p_1/p_2) for given $p_1$and p_2. Hence the new budget line shifts parallely downwards subject to the fact that after the shift, it is tangent to the previous indifference curve. The consumer can attain the same level of utility and the real income remains constant in terms of utility after adjusting money income and utility is also maximised. After adjustment of money income, budget line is A'C' along which real income in terms of utility remains constant after change in p_1 for given p_2. This budget line is known as compensated budget line. Under such budget line equilibrium will necessarily take place at point e_1'. Hence under constant real income in terms of utility, as $p_1$decreases for given p_2, x_1 increases (from x_1^0 to x_1') by substituting x_2 (from x_2^0 to x_2^1). This is known as own price substitution effect for x_1 which is negative and indifference curve is downward sloping strictly convex to the origin. But as x_1 increases from x_1^0 to x_1^1 and real income also increases, the demand for good I increases from x_1^0 to x_1' through a rise in real income. This would indicate that by income effect for a price change, x_1 is a normal good. Clearly, we have own price effect consists of own substitution effect and income effect for a price change, where own substitution effect in negative but income effect for a price change can take any algebrical sign depending on the good is normal, superior or inferior.

Mathematical Presentation

We already know from the first order conditions of utility Maximisation that,

$$\frac{dL(x_1,x_2)}{dx_1} = \frac{dU(x_1,x_2)}{dx_1} - \lambda p_1 = 0 \text{ ------------- (a)}$$

$$\frac{dL(x_1,x_2)}{dx_2} = \frac{dU(x_1,x_2)}{dx_2} - \lambda p_2 = 0 \text{ ------------- (b)}$$

$$\frac{dL(x_1,x_2)}{d\lambda} = M - p_1x_1 - p_2x_2 = 0 \text{ ------------- (c)}$$

We then totally differentiate these equations and get:

$U_{11}\, dx_1 + U_{12}\, dx_2 - p_1\, d\lambda = \lambda\, dp_1$ ------------- (e)

$U_{21}\, dx_1 + U_{22}\, dx_2 - p_2\, d\lambda = \lambda\, dp_2$ ------------- (f)

$-p_1\, dx_1 - p_2\, dx_2 + 0\, .d\lambda = -dM + x_1\, dp_1 + x_2\, dp_2$ ------------ (g)

By using Cramer's rule we have,

$$dx_1 = \begin{pmatrix} \lambda dp_1 & U_{12} & -p_1 \\ \lambda dp_2 & U_{22} & -p_2 \\ -dM + dp_1 + dp_2 & -p_2 & 0 \end{pmatrix} / |D|$$

where, $|D| = \begin{pmatrix} U_{11} & U_{12} & -p_1 \\ U_{21} & U_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{pmatrix}$ and $U_{ij} = \dfrac{\partial^2 U(x_i,x_j)}{\partial x_i \partial x_j}$.

Or, we can write,

$$dx_1 = \frac{\lambda dp_1 D_{11} + \lambda dp_2 D_{21} + (-dM + x_1 dp_1 + x_2 dp_2) D_{31}}{|D|} \text{ ------------- (h),}$$

where D_{ij} is the co-factor of the i^{th} row and j^{th} column of the determinant |D|. For income effect we know dp_1=dp_2=0, therefore we have from equation (h),

$$\partial x_1 = \frac{-D_{31}\partial M}{|D|}, \text{ or } \left(\frac{\partial x_1}{\partial M}\right)_{\bar{p}} = \frac{p_2U_{12} - p_1U_{22}}{|D|} \text{ ------------- (i)}$$

Now for own price effect we have dM=dp_2=0. So from equation (h) we get,

$$dx_1 = \frac{\lambda D_{11}\partial p_1 + x_1 D_{31}\partial p_1}{|D|} \text{ or, } \left(\frac{dx_1}{\partial p_1}\right)_{\bar{M},\bar{p}_2} = \frac{\lambda D_{11}}{|D|} + x_1\frac{\lambda D_{31}}{|D|} \text{ ------------- (j)}$$

Lastly, to find out own substitution effect we consider utility is constant in terms of income so, $-dM+x_1dp_1+x_2dp_2=0$ and $dp_2=0$. We have from equation (h), $(\frac{dx_1}{\partial p_1})_{\bar{U},\bar{p}_2} = \frac{\lambda D_{11}}{|D|}$ ------------- (k).

Therefore, from equation (i), (j) and (k), we get,

$(\frac{\partial x_1}{\partial x_1})_{\bar{M},\bar{p}_2} = (\frac{\partial x_1}{\partial p_1})_{\bar{U},\bar{p}_2} - x_1(\frac{\partial x_1}{\partial M})_{\bar{p}}$, which is the Slutsky's equation.

1.5.6 Compensated Demand Curve

Compensated demand function for a commodity (say x_1) of an individual consumer represents demand quantity for that good (which is purchased by the consumer) as a function of price of that good and prices of other goods under constant real income and constant other things.

Notationaly, it is given by $x_1=x_1(p_1, p_2, y)$, where y is the real income.

Demand curve for a good showing the relationship between demand quantity for that good and its own price given other things and given real income is known as compensated demand curve along which real income is constant (real income is defined by the ratio between money income and price level). Along the demand curve price of that good changes, so money income should be proportionately adjusted or compensated such that real income is constant. That is why the corresponding demand function and demand curve is known as compensated demand function and compensated demand curve.

There are two different approaches to the measurement of real income, viz.,

- Hicksian Approach: In Hicksian approach, real income is measured in forms of utility. A constant real income means a constant utility. Thus, demand quantity for a good purchased by a consumer as a function of prices of all goods under constant utility and constant other things is known as compensated Hicksian demand function.

  Demand curve for a commodity showing the relationship between quantity demand for that commodity and it's own price under constant other things and constant real income in terms of utility is known as compensated Hicksian demand curve.

- Slutsky's Approach: In this approach, real income is measured in terms of purchasing power. A constant real income means a constant purchasing power (it is denoted by y_p). Demand quantity for a good purchased by a consumer as a function of prices of all goods under constant other things and constant purchasing power is known as compensated Slutsky's demand function and corresponding demand curve is known as compensated Slutsky's demand curve.

  Below we discuss the Hicksian approach graphically.

Derivation of compensated demand curve:

Hicksian compensated demand function for x_1 is given by $x_1=x_1(p_1, p_2, U)$, where Hicksian compensated demand curve for a good represent the relationship between price of that good with its own demand quantity for given prices of other goods and real income in terms of utility.

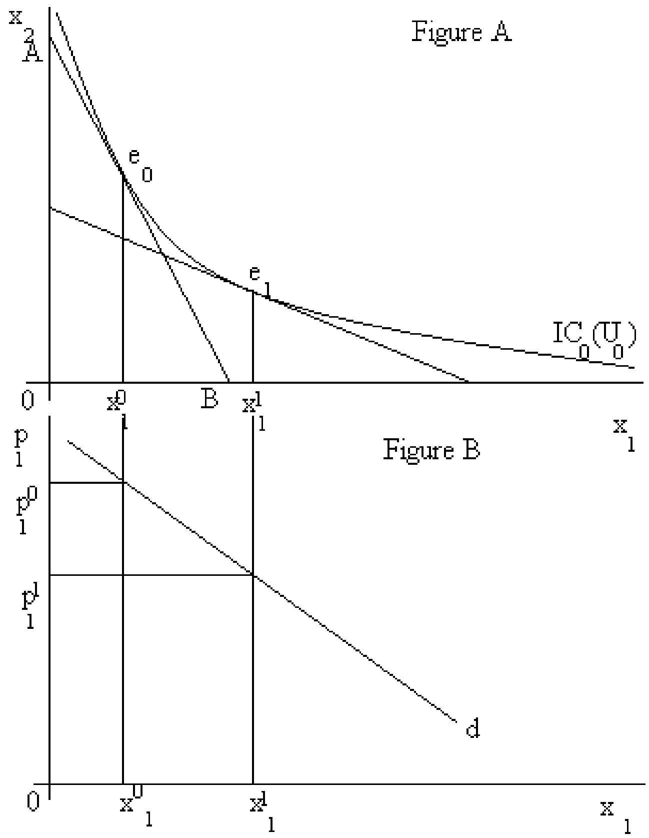

Fig. 1.7: Derivation of Compensated Demand Curve

We now derive this graphically. Suppose, initial equilibrium is attained at e_0 in Figure A where price of good on is p_1^0 and price of good two is p_2^0 respectively and utility is fixed at U_0. Corresponding indifference curve is IC_0. Compensated Hicksian demand for x_1 is at x_1^0. Expenditure line is AB at initial equilibrium with absolute slope p_1^0/p_2^0.

Plot this x_1^0 and p_1^0 in Figure B. Suppose, for given utility and p_2, p_1 decreases to p_1^1. Therefore, absolute slope of the budget line decreases, i.e., expenditure line become flatter. Since utility is constant, the indifference curve remains the same as before. Therefore, expenditure is minimised for given utility at point e_1 in Figure A, as indifference curve is downward sloping strictly convex to the origin. So compensated Hicksian demand for good I increases to x_1^1 plot p_1^1 and x_1^1 in Figure B. By joining all such pair of p_1 and x_1 in Figure B, we have a downward sloping curve in p_1-x_1 plane, for given p_2 and utility. This downward sloping demand curve is the Hicksian compensated demand curve. This is shown in the above Figure B.

Check Your Progress 3

1) Define compensated demand curve in one sentence. What are measured in the axes of the figure to draw a compensated demand curve in Hicksian approach?

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2) What is the sign of the slope of the compensated demand curve? Can the compensated demand curve take positive slope?

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3) What is the main difference between Hicksian approach and Slutsky's approach regarding compensation in the context of compensated demand curve?

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1.6 LET US SUM UP

In this unit, we discussed various aspect of consumer behaviour theory. We elaborated two classical theories (viz. Cardinal Approach and Ordinal Approach). In ordinal approach discussing the indifference curve theory we show that indifference curve in general is downward sloping and strictly convex to the origin. Consumer equilibrium in ordinal approach was found out both graphically and algebrically. In the ordinal approach at equilibrium two condition must satisfied. The first condition is the equality between slope of the indifference curve and slope of the budget line, which indicates that at equilibrium slope of the budget line must be equal to slope of the indifference curve. The second condition shows that at equilibrium budget constraint must satisfy with equality sign, i.e., consumer spends all her income in consumption. This condition is derived from the assumption of non-satiation of all goods. The income effect and substitution effect have been presented and meanings explained. An income effect is the change in consumer demand due to unit change in income when other things are held constant and substitution effect is the change in consumer demand due to change in prices of any one good, the utility and other things remaining unchanged. In the next section, we discussed the Slutsky's theorem, which is the relationship between price effect, income effect and substitution effect. It shows that price effect is the sum of substitution effect and income effect. Finally, we discussed the compensated demand curve analysis derived by Hicks.

1.7 KEY WORDS

Compensated Demand Curve: The graph showing the relationship between the price of a good and quantity consumed, if the real income is held constant.

Demand Curve: Amount of a good consumers are willing purchase as a function of its price.

Income Effect: Increased consumption brought about by an increased income when the prices of goods are held constant.

Indifference Curve: A graphical representation of different combinations of goods yielding the same level of satisfactions.

Revealed Preference: Determination of preferences of a consumer through observation of her choices.

Slutsy Equation: A mathematical formulation that separates substitution and income effects of a price change on utility-maximising choices.

Substitution Effect: Reflection of a situation when consumption changes due to a change in price while the level of satisfaction is held constant.

Utility: Satisfaction derived by a consumer from consumption of a good.

1.8 SOME USEFUL BOOKS

J. Henderson & Richard E. Quandt (2003), *Microeconomic Theory: Mathematical Approach*, Tata McGraw-Hill Publishing Company Limited, New Delhi.

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillian.

Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York.

1.9 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) See section 1.3.1

2) At consumer equilibrium $\frac{dU(x)}{dx} - \lambda p_x = 0$ must hold, we have $\frac{dU(x)}{dx} =$ 1/x, λ=5 and p_x = 2, therefore at equilibrium 1/x = 10 or, x^* = 1/10.

Second order condition is satisfied because $\frac{\partial^2 U(x)}{\partial x^2} = -1/x^2 < 0$ at equilibrium.

Check Your Progress 2

1) See section 1.5.1

2) Lagrange function is given by

$L(x_1, x_2) = 10x_1^{0.3}x_2^{0.7} + \lambda\,(200 - 5x_1 - 2x_2)$

$\frac{dU(x_1, x_2)}{dx_1} = 3x_1^{-0.7}\,x_2^{0.7}$ and $\frac{dU(x_1, x_2)}{dx_2} = 7x_1^{0.3}x_2^{-0.3}$, therefore

$\frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} = p_1/p_2$ imply $3/7x_1^{-0.4}x_2^{0.4} = 5/2$, or $x_1 = (6/35)^4 x_2$.

3) Lagrange function is given by

$L(x_1, x_2) = 10x_1^{0.5}x_2^{0.5} + \lambda\,(100 - 2x_1 - 4x_2)$,

$\frac{dU(x_1, x_2)}{dx_1} = 5x_1^{-0.5}\,x_2^{0.5}$ and $\frac{dU(x_1, x_2)}{dx_2} = 5x_1^{0.5}x_2^{-0.5}$, therefore

$\frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} = p_1/p_2$ imply $x_1^{-1}x_2 = 1$, or $x_1 = x_2$ ------ (I).

$M = p_1x_1 + p_2x_2$ implies $100 = 2x_1 + 4x_2$ ---------- (II).

Solving equation (I) and (II), we get $(x_1^*, x_2^*) = (50/3, 50/3)$.

Check Your Progress 3

1) See sub-section 1.5.6

2) Compensated demand curve is always downward sloping, it can't have positive slope in any occasion.

3) See sub-section 1.5.6