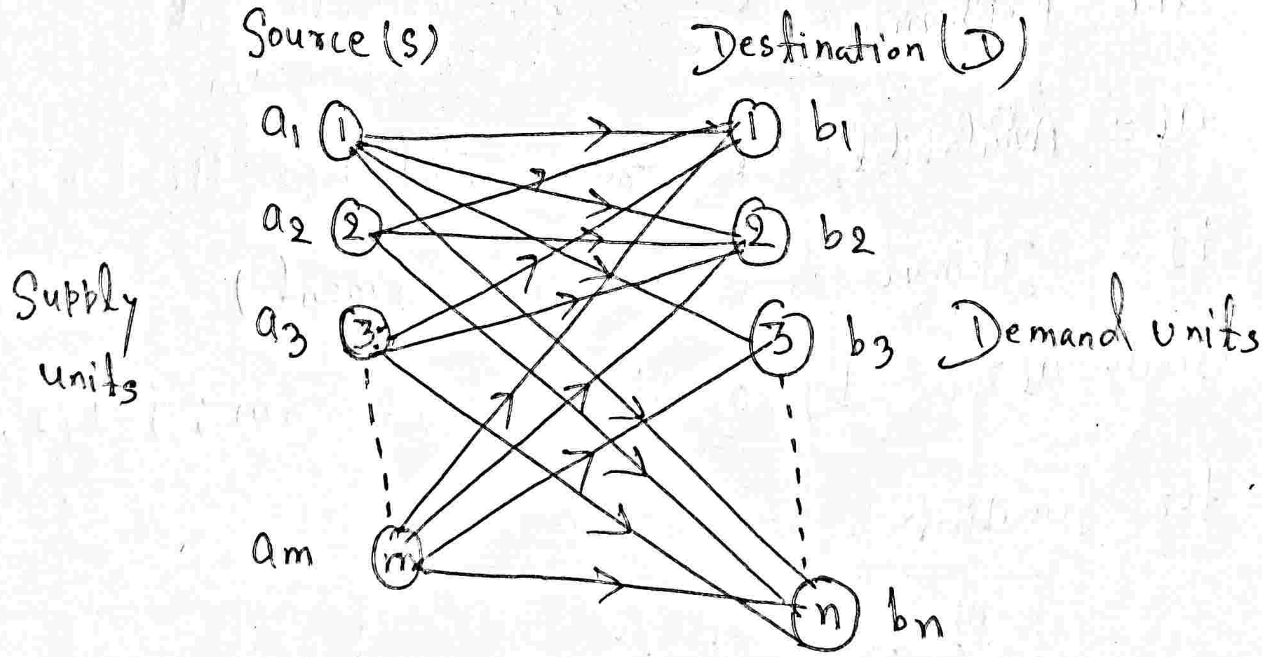
Transportation Problem

Introduction to Transportation Problem (T.P)

T.P is one of the most useful Operation Research Problem and is a special case of LPP.

Various quantities of single homogeneous commodity are initially stored at various origin. The basic objective of T.P is to transport these commodities from various origin to different destination such that total transportation cost is minimum.

General formulation:



* Nodes represent Source and Destination

* Arcs (i, j) represent the amount x_{ij} shipped from i^{th} Source to j^{th} destination with shipment cost as C_{ij}

* The amount of supply at source i is a_i and amount of demand at destination j is b_j.

T.P model is based upon the assumption of balancing i.e, Total demand = Total Supply.

If a given T.P is not balanced, one can always add dummy source or dummy destination to balance the problem.

a_i = Availability (Row requirement)

b_j = Demand (column requirement)

Clearly $a_i \geq 0$, $b_j \geq 0$ $\forall$ $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots n$

for feasibility, we assume

$$\boxed{\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$$

Let C_{ij} = cost of shipping one unit from source S_i to destination D_j

X_{ij} = quantity shipped from source S_i to distination D_j.

In table form, we can write

	D_1	D_2	...	D_n	
S_1	C_{11}	C_{12}		C_{1n}	a_1
S_2	C_{21}	C_{22}		C_{2n}	a_2
$\vdots$					
S_m	C_{m1}	C_{m2}		C_{mn}	a_m
	b_1	b_2		b_n ← Demand	

$a_1, a_2, \ldots, a_m$ } Availability.

Solution of the above T.P taken the form

$X =$

X_{11}	X_{12}		X_{1n}
X_{21}	X_{22}		X_{2n}
X_{m1}	X_{m2}		X_{mn}

, $x_{ij} \geq 0 \ \forall i, j$.

$$\sum_{j=1}^{n} X_{ij} = a_i \ , \ i = 1, 2, \ldots, m \longrightarrow (1)$$

$$x_{11} + x_{12} + \cdots + x_{1n} = a_1$$

$$x_{21} + x_{22} + \cdots + x_{2n} = a_2$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$x_{m1} + x_{m2} + \cdots + x_{mn} = a_m .$$

Similarly, $\sum_{i=1}^{m} x_{ij} = b_j , \; j = 1, 2, \ldots, n \longrightarrow (2)$

$$x_{11} + x_{21} + \cdots + x_{m1} = b_1$$

$$x_{12} + x_{22} + \cdots + x_{m2} = b_2$$

$$x_{1n} + x_{2n} + \cdots + x_{mn} = b_n .$$