**TOPIC: The Explanation of Stationary waves:**

When the progressive waves traveling through a medium are incident normally on rigid boundary, they are reflected with a phase change of $\pi$ radian. The reflected waves have the same amplitude & wavelength as the incident waves, but they travel in opposite direction. The medium contain two identical wave-train traveling same path but in opposite direction. It gives rise to stationary waves or standing waves.

Stationary waves are produced when two exactly identical progressive waves (having the same amplitude, wavelength & same speed) traveling through a medium along the same path in exactly opposite directions, interfere with each other.

Note:- At a rigid boundary or a closed end, the progressive waves are reflected with a phase reversal, while at an open boundary or open end the waves are reflected without any change of phase.

**The boundary condition of finite medium:**

i. In an unbounded (infinite) medium, the wave travels in a given direction continuously until it gets dissipated.

ii. In a bounded (finite) medium by rigid boundary the wave travels in a given direction, and arrives at a rigid boundary and gets reflected from it.

iii. When such wave is reflected from a rigid boundary there is reverse of phase, (phase difference is $\pi$ radian).

iv. If the wave travelling through a denser medium arrives at the surface of a rarer medium, after reflection, direction of its wave velocity is reversed but direction of particle velocity does not change, there is no phase difference between incident wave and reflected wave.

v. When a wave is travelling through an air column inside a pipe closed at one end, boundary condition shows that closed end is a rigid boundary and open end acts as interface between open and closed end of pipe.

vi. When wave travels through a pipe and reflected from closed end then it is reflected with a reverse of phase. (Phase difference is $\pi$ radian) vii. When wave travels through a pipe open at both ends. Open end acts as the interface between a boundary of bounded medium and unbounded medium. Therefore the wave is reflected at each open end without any change of phase.

viii. Wave reflected from boundary of rigid surface, has phase difference of $\pi$ radian and wave reflected from unbounded medium, has phase difference of zero radian

**Introduction of stationary waves:**
Two identical progressive wave (transverse or longitudinal) travelling along the same path in opposite direction, they interfere to each other, by superposition of waves resultant wave is obtained in the form of loops are called stationary waves.

**Que** Explain the formation of stationary wave by analytical method. What are nodes and antinodes? Show that the distance between two successive nodes or antinodes is or

**Explain the formation of stationary wave by analytical method. Show that nodes and antinodes are equally spaced in

**Ans:** Two exactly identical transverse waves traveling in opposite directions along the same path can be expressed by the equations,

\[
y_1 = \sin 2\pi \left[ \frac{t}{T} - \frac{x}{\lambda} \right] = \sin 2\pi \left[ nt - \frac{x}{\lambda} \right]
\]

\[
y_2 = \sin 2\pi \left[ \frac{t}{T} + \frac{x}{\lambda} \right] = \sin 2\pi \left[ nt + \frac{x}{\lambda} \right]
\]

Both wave have same amplitude \( a \) the same wavelength \( \lambda \) & same frequency \( n \) the resultant displacement is given by,

\[
Y = y_1 + y_2
\]

\[
= \sin 2\pi \left[ nt - \frac{x}{\lambda} \right] + \sin 2\pi \left[ nt + \frac{x}{\lambda} \right]
\]

Using the trigonometric relation

\[
sin C + sin D = 2 \sin \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right)
\]

We can write,

\[
y = 2\sin(2\pi nt) \cos(-2\pi nt)
\]

Since \( \cos(-\theta) = \cos\theta \)

\[
y = 2\sin(2\pi nt) \cos(2\pi nt)
\]

\[
y = A\sin(2\pi nt)
\]
Where \( A = 2a \) is the amplitude of resultant motion. This equation represents resultant motion is S.H.M. but does represent simple harmonic progressive wave, since does not contain a term

\[
\cos \left( 2\pi \frac{x}{\lambda} \right)
\]

The absence of the term \( x/\lambda \) in above equation shows that the resultant waives not move in the forward or backward direction hence it is called as stationary or standing waves.

**Position antinodes:** The amplitude of the stationary is variable & depends on the position \( X \) of the particle. As \( A = \frac{2\pi x}{\lambda} \)

\[
2a \cos \left( \frac{2\pi x}{\lambda} \right)
\]

The amplitude is maximum

\[
i.e. \quad A = \pm 2a \quad i.e. \quad \cos \left( \frac{2\pi x}{\lambda} \right) = \pm 1
\]

i.e. where \( x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi \),

i.e. where \( x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \frac{\lambda}{2} \),

the particle situated at these, white vibrate with maximum amplitude. **These points are called as antinodes.**

the distance between two successive antinodes or maxima is \( \frac{\lambda}{2} - 0 = \frac{\lambda}{2} \),

\[
\lambda - \frac{\lambda}{2} = \frac{\lambda}{2}
\]

\[
3 \frac{\lambda}{2} - \lambda = \frac{\lambda}{2}
\]

i.e. distance between two successive antinodes or maxima is \( \lambda /2 \).

**Position of nodes:** The amplitude is minimum i.e. \( A = 0 \) as \( 2a \cos \left( \frac{2\pi x}{\lambda} \right) = 0 \).
where \[
\frac{2\pi x}{\lambda} = 0
\]
i.e. when \[
\frac{2\pi x}{\lambda} = \frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}
\]
i.e. where \(x = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}\)

**The distance between two successive minima**

\[
= 3\frac{\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2},
\]
\[
5\frac{\lambda}{4} - 3\frac{\lambda}{4} = \frac{\lambda}{2}, 7\frac{\lambda}{4} - 5\frac{\lambda}{4} = \frac{\lambda}{2}.
\]

Nodes are the points in the medium at which, the displacement of the particle is always zero called as nodes. The distance between any two successive nodes is \(\lambda/2\). Distance between adjacent antinodes and nodes is \(\lambda/4\).

***Displacement node and antinodes:

The point at which the intensity becomes zero are called the displacement nodes. On the other hand, the point at which the intensity becomes to maximum are called as displacement antinodes.

In a longitudinal stationary wave, it can be shown that there are no pressure changes at the displacement antinodes, therefore, displacement antinodes are also referred to as pressure nodes.

At node the particles are permanently at rest. However, pressure at these points varies between maximum and minimum values. Therefore, displacement nodes are pressure antinodes.

***The characteristics of stationary waves:

1. Stationary waves are produced due to the interference between two exactly identical waves traveling through the medium in opposite directions.
2. There are some points in the medium at which the displacement is zero. Such points are called as nodes.
3. When stationary waves are set up in the medium, there are some points at which the amplitude is maximum are called as antinodes.
4. The distance between two successive nodes or antinodes is equal to \(\lambda/2\).
5. The distance between node & adjacent antinodes is \(\lambda/4\).
(6) In the stationary waves, all the particles, except nodes, vibrates with same period as that of the interfering waves. The amplitude of vibrations increases from node to antinodes.

(7) All the particles in one loop are in same amplitude of vibrations increases from node to antinodes.

(8) All the particles in one loop are in same phase while particles in successive loops are in out of phase.

(9) Stationary waves are periodic in space & periodic in time.

(10) Stationary waves are produced by superposition of two progressive waves with resultant velocity is zero, hence the waves cannot be propagated through the medium i.e. they do not transfer the energy through medium. Que. 6 : Distinguish between progressive waves and stationary waves. Ans:-

<table>
<thead>
<tr>
<th>Progressive Wave</th>
<th>Stationary Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. These waves are continuously away from the source.</td>
<td>1. These waves do not move in any direction.</td>
</tr>
<tr>
<td>2. Phase changes progressively in one wavelength</td>
<td>2. All the particles in one loop are in same phase while particles in adjacent loops are out of phase</td>
</tr>
<tr>
<td>3. There is net transfer of energy in the medium.</td>
<td>3. There is no transfer of energy in the medium.</td>
</tr>
<tr>
<td>4. These waves are produced when the disturbance is created in the medium.</td>
<td>4. These waves are produced due to interference between two exactly identical waves travelling in opposite direction along same state line.</td>
</tr>
</tbody>
</table>

**Harmonic and Overtone**

*Harmonic* - The integral multiple of fundamental frequency is called its harmonic.
The fundamental frequency \( (n) \) is called first harmonic, then \( 2n \) be the second harmonic, \( 3n \) be the third harmonic, and so on.

Overtones - vibration of higher frequency which are actually present in addition to the fundamental frequency are called overtones.

***Distinguish between harmonic and overtone:

<table>
<thead>
<tr>
<th>Harmonics</th>
<th>Overtones</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The fundamental frequency and all its integral multiples are called as harmonics</td>
<td>1. The frequency which is immediately higher than the fundamental frequency is called as overtone.</td>
</tr>
<tr>
<td>2. Fundamental frequency itself is first harmonic</td>
<td>2. The frequency which is immediately higher than the fundamental frequency is called first overtone.</td>
</tr>
<tr>
<td>3. In some vibration harmonic may be absent</td>
<td>3. All overtone always present</td>
</tr>
<tr>
<td>4. All harmonic are not necessarily overtones.</td>
<td>4. All overtone are harmonic.</td>
</tr>
</tbody>
</table>

**Vibration On String:**

When a stretched string is plucked at the some point, perpendicular to its length a transverse waves are produced along the string. The velocity of the transverse waves depends on the tension \( (T) \) applied to the string & the mass per unit length \( (m) \) of the string. The velocity of the transverse waves along the string is given by

\[
V = \sqrt{\frac{T}{m}}
\]

*** Que. Explain the different mode of vibrating string. Obtain the expression for frequency in each case.

Ans:-
i. When the transverse waves reach the fixed ends of the string, they get reflected with the phase change of \( \pi \) radian. The reflected waves interfere with the incident & produced the stationary wave along the string. **The fixed end of the string act as nodes i.e. point of zero displacement.** The different ways in which the string can vibrate are called as modes of vibrations of the string.

ii. **First mode of vibration:** If the string is stretched between two rigid supports and plucked at the centre, antinodes is formed at the centre and string is vibrates as shown in fig. a. this is the simplest mode of vibration in the string, called as fundamental mode of vibration of string.

iii. If \( \lambda \) is the wavelength & \( l \) be length of the string, length of the loop = \( \lambda / 2 = l \)

\[ \lambda = 2l \]

If \( n \) is the frequency of vibrations of the string,

\[ n = \frac{v}{\lambda} = \frac{1}{\lambda} \sqrt{\frac{T}{m}} \]

\[ \therefore n = \frac{1}{2l} \sqrt{\frac{T}{m}} \] \hspace{1em} (1)
This is the lowest frequency with which the string can vibrate, called as **fundamental frequency of vibration or first harmonic.**

**iv. Second mode of vibration:** The string vibrates with two loops, with three nodes & two antinodes, as shown fig. b If $\lambda_1$ is wavelength of vibrations,

v. Length of the one loop $= \lambda_1 = \ell$

Therefore corresponding frequency ($n_1$) of vibrations is given by

$$n_1 = \frac{1}{\lambda_1} \sqrt{\frac{T}{m}}$$

It can be seen that $n_1 = 2n$

**The frequency of vibrations is called as second harmonic or the first overtone.**

vi. **Third mode of vibration:** The string is plucked such that it vibrates with three loops, with four nodes & three antinodes as shown in fig. If $\lambda_2$ is the wavelength

The length of one loop $= \frac{\lambda_2}{2} = \frac{\ell}{3} \Rightarrow \lambda_2 = \frac{2\ell}{3}$ therefore the frequency of vibrations $n_2$ is given as

$$n_2 = \frac{1}{\lambda_2} \sqrt{\frac{T}{m}}$$

$$n_2 = \frac{3}{2\ell} \sqrt{\frac{T}{m}}$$

$\therefore n_2 = 3n$

**This frequency of vibrations is called as third harmonic or second overtone.**

vii. Above consideration show that the modes of vibrations of vibrations of stretched string consist of the fundamental frequency along with all integral multiples. e.g. $n$, $2n$, $3n$, $4n$, $5n$ --------

**Remark:**

1. If $n_p$ is the frequency of $p^{th}$ overtone then

$$n_p = \frac{(P + 1)}{2\ell} \sqrt{\frac{T}{m}}$$
2. The \( P^{th} \) overtone is \((P+1)^{th}\) harmonic.

3. Frequency of \( P^{th} \) overtone = \((P+1)\) \( n \)

***Que. : If wire of density \( \rho \) and Young's modulus \( Y \) is stretched between two bridges \( L \) unit apart, show that fundamental frequency of the wire is in

\[
\frac{n}{2L} = \sqrt{\frac{Y\ell}{\rho L}}
\]

Where \( \ell \) Extension in the wire.

Ans : \( M = (\pi r^2)\rho \) and \( m = \frac{M}{L} = \pi r^2 \rho \)

The stress in the wire = \((\pi r^2)\rho \)

\[
\therefore \frac{T}{m} = \frac{T}{(\pi r^2)\rho} = \frac{\text{Stress}}{\rho}
\]

Thus the fundamental frequency of vibration of wire,

\[
\frac{n}{2L} = \sqrt{\frac{\ell}{\rho L}}
\]

As \( \ell \) is elastic extension of wire under tension \( T \), strain = \( \frac{\ell}{L} \)

Young's modulus, \( Y = \frac{\text{Stress}}{\text{strain}} \)

\[
\text{Stress} = Y \times \text{strain} = Y \frac{\ell}{L}
\]

\[
\frac{n}{2L} = \sqrt{\frac{Y\ell}{\rho L}}
\]

Que.10: If wire of density \( \rho \) and Young's modulus \( Y \) is stretched between two bridges \( L \) unit apart, show that fundamental frequency of the wire is in

\[
\frac{n}{2L} = \sqrt{\frac{Y\alpha L}{\rho}}
\]

Where \( \alpha \) coefficient of linear expansion.
Ans: $M = (\pi r^2)L\rho$ and $m = \frac{M}{L} = \pi r^2 \rho$

The stress in the wire = $\frac{T}{(\pi r^2)\rho}$

$\therefore \frac{T}{m} = \frac{T}{(\pi r^2)\rho} = \frac{\text{Stress}}{\rho}$

Thus the fundamental frequency of vibration of wire,

$n = \frac{1}{2L} \sqrt{\frac{T}{m}}$

$n = \frac{1}{2L} \sqrt{\frac{\text{Stress}}{\rho}}$

Where stress = strain $\times$ $Y$

But strain = $\frac{\xi}{L} = \alpha t$

Stress = $Y\alpha t$

$n = \frac{1}{2\ell} \sqrt{\frac{Y\alpha t}{\rho}}$

**The law of vibrating string:**

Laws of vibrating strings - The fundamental frequency of vibration of the string under tension is given by

$n = \frac{1}{2L} \sqrt{\frac{T}{m}}$

This relation is used to formulate three laws of the string

**1. First law of string (law of length):** The fundamental frequency of vibrations is inversely proportional to the vibrating length of the string, if tension & mass per unit length are kept constant
If \( T \) & \( m \) are kept constant, i.e \( n \ell = \text{constant}, \)

\[ n_1 \ell_1 = n_2 \ell_2 = k \]

(2) \textbf{Law of tension:} - The fundamental frequency of vibrations is directly proportional to the square root of the tension in the string, if the vibrating length \& mass per unit length of the string.

i.e. \( n \propto \sqrt{T} \), if \( \ell \) \& \( m \) are constant.

(3) \textbf{Law of mass per unit length:} - The fundamental frequency of vibrating string is inversely proportional to the mass per unit length if \( T \) \& \( \ell \) are constant.

\[ n \propto \sqrt{m} \], if \( T \) \& \( \ell \) are constant.

\[ n\sqrt{m} = \text{constant} \]

\[ n_1\sqrt{m_1} = n_2\sqrt{m_2} = \text{constant} \]

\[ m = \frac{M}{\ell} = \frac{\rho V}{\ell} = \frac{\rho \pi r^2 \ell}{\ell} = \rho \pi r^2 \]

\[ n_1\sqrt{\rho \pi r^2_1} = n_2\sqrt{\rho \pi r^2_2} \]

( if the material is constant \( \rho_1 = \rho_2 = \rho \))

\[ n_1 r_1 = n_2 r_2 \]

\[ n \propto \frac{1}{r} \]

If the material of string are different \((\rho_1, \rho_2)\) then

\[ n \propto \frac{1}{\sqrt{\rho}} \]

Que.12: If \(?\) is specific gravity or Relative density then showing that \( \frac{n_1}{n_2} = \sqrt{\frac{\sigma}{\sigma-1}} \)

Ans:

Specific gravity (\(?\)) : It is ratio of density of substance to density of water.

OR

It is the ratio of weight of body in air to the loss of weight of body in water.
\[ \sigma = \frac{\text{loss of weight of body in water}}{\text{weight of body}} \]

\[ \sigma = \frac{\text{weight of body in air} - \text{weight of body in water}}{\text{weight of body}} \]

\[ \sigma = \frac{T_1}{T_1 - T_2} \]

\[ \frac{1}{\sigma} = \frac{T_1}{T_1 - T_2} \]

\[ \frac{1}{\sigma} = 1 - \frac{T_2}{T_1} \]

\[ \frac{T_2}{T_1} = 1 - \sigma \]

\[ \frac{T_1}{T_2} = \frac{\sigma}{\sigma - 1} \]

\[ n_1 = \sqrt{\sigma} \]

\[ n_2 = \sqrt{\frac{T_1}{T_2}} \]

\[ \frac{n_1}{n_2} = \sqrt{\frac{\sigma}{\sigma - 1}} \]

\[ \frac{n_1}{n_2} = \sqrt{\frac{\sigma}{\sigma - 1}} \]

**Que.** show that only the odd harmonics are present in the vibration of air column in a pipe, closed at one end.

**Ans:** -

i. Boundary condition: When the stationary waves are produced in a pipe, closed at one end. A node is formed at the closed end and antinodes formed at the open end.

Different modes of vibration are shown in the following fig.
ii. **First mode of vibration:** Let $l$ be the length of the pipe. The simplest mode of vibration in which the air column can vibrate is shown in fig. a, with one antinode at open & one node at the closed end. If $\lambda$ is the wavelength of the waves.

Length of air column = distance between node and antinodes as shown in fig.

\[
\begin{align*}
\lambda &= \frac{\lambda}{4} \\
\lambda &= 4l
\end{align*}
\]

If $V$ the velocity of sound waves, $V = n\lambda$

\[
\left\{\begin{array}{l}
n = \frac{V}{4l} \quad \text{(1)}
\end{array}\right.
\]

This is the lowest frequency with which the air column can vibrate, called the fundamental frequency or first harmonic.

iii. **Second mode of vibration:** The next mode of vibration is shown in fig. b. If $n_1$ & $\lambda_1$ are the frequency & wavelength respectively, the length of the air column is given as

\[
l = 3\frac{\lambda_1}{4}
\]

\[
\lambda_1 = \frac{4l}{3}
\]

\[
V = n_1\lambda_1 = n_1 \times \frac{4l}{3}
\]

\[
n_1 = \frac{3V}{4l} = 3n \quad \text{(2)}
\]
This frequency of vibration is called **third harmonic or first overtone**.

iv. **Third mode of vibration:** The next mode of vibration is shown in fig. c if \( n_2 \) be the frequency & \( \lambda_2 \) wavelength of sound waves. The length of the air column is given as,

\[
\ell = \frac{5\lambda_2}{4}
\]

\[
\lambda_2 = \frac{4\ell}{5}
\]

The velocity of sound is \( V = n_2 \lambda_2 \)

\[
n_2 = \frac{5V}{4\ell}
\]

\[
n_2 = 5n \------------------------- (3)
\]

This frequency of vibration is called **as fifth harmonic or second overtone**.

Thus air column in a pipe closed at the one end vibrates with only odd harmonics. e.i. \( n, 3n, 5n, 7n \----------

1

1

The frequency of the \( P^{th} \) over tone is equal to

\( (2P+1) n_0 \), Where \( n_0 = \frac{V}{4\ell} \)

Que.14 : Show that all the harmonic are present in the vibration of all column in a pipe open at both the end.

Ans:

i. **Boundary condition:** When the stationary waves are produced in a pipe open at both the ends the antinodes are formed at the open ends.

Different mode of vibration as shown in figure, in each case \( \ell \) be the length of air column pipe
ii. **First mode of vibration**: The simplest mode of vibration in which the air column can vibrate is shown in fig. a, with antinodes formed at both open ends.

Let \( n \) and \( \lambda \) be the frequency and wavelength for fundamental mode of vibration.

The length of the air column is equal to the distance between two successive antinodes

\[
\therefore \ell = \frac{\lambda}{2} \quad \& \quad \lambda = 2\ell
\]

\[
\therefore \text{Fundamental frequency (} n \text{)} = \frac{V}{2\ell}
\]

This is the lowest frequency with which the air column can vibrate, called as fundamental frequency or first harmonics

iii. **Second mode of vibration**: The next mode of vibration is shown in fig. 2, the length of the air column is given as,

\( l = \lambda_1 \) where \( \lambda_1 \) is the wavelength of vibration & \( n_1 \) is the frequency of vibration

\[
n_1 = \frac{V}{\lambda_1} = \frac{V}{\ell}
\]

\[
n_1 = 2n \quad \text{(2)}
\]

This frequency of vibration is called as second harmonic or first overtone.

iv. **Third mode of vibration**: The next mode of vibration is shown in fig. 3, let \( n_2 \) be the frequency & \( \lambda_2 \) be the wavelength, the length of the air column is
This frequency of vibration is called third harmonic or second overtone.

This vibration in a pipe open at both end consist of fundamental frequency along with all integral multiple i.e. \( n, \ 2n, \ 3n, \ 5n, \ldots \) \[ \text{the frequency of first overtone is} \ 2n \ \text{that of second overtone is} \ 3n. \text{Thus all harmonic are present.} \]

In general the frequency of \( p^{th} \) over tone is \( (p+1) n \) where \( n = \frac{V}{2\ell} \)

(\text{It may be noted that the frequency of the fundamental note of an open pipe is double than that of open pipe is double than that for closed organ pipe of same length in air (}n_o = 2n_1\text{) })

\textbf{End correction and it's cause:}

The antinodes is not situated exactly at the tip of the open end but a little a above it. This distance is called as the end correction is equal to 0.3d where d is inner diameter of the tube.

\[ \therefore L = \ell + 0.3d \]

In fundamental mode, pipe closed at one end

\[ V = n = 4n_1 \ell \]

\[ V = 4n(\ell + 0.3d) \quad (2) \]

\text{This the formula for velocity of sound in air}

\textbf{Que.16: Show that for pipe closed at one end}

\[ e = \frac{n_1 \ell_1 - n_2 \ell_2}{n_2 - n_1} \]

\text{Ans: If} \ n_1 \text{ and} \ n_2 \text{ is frequencies then velocity of sound for pipe closed at one end is}

\[ v = 4n_1L_1 = 4n_2L_2 \]

\[ n_1L_1 = n_2L_2 \]
\[ n_1(\ell_1 + e) = n_2(\ell_2 + e) \]

\[ \therefore n_1\ell_1 - n_2\ell_2 = e(n_2 - n_1) \]

\[ \therefore e = \frac{n_1\ell_1 - n_2\ell_2}{n_2 - n_1} \]

Que.16: Show that for pipe open at both end the end correction

\[ e = \frac{n_1\ell_1 - n_2\ell_2}{2(n_2 - n_1)} \]  

Ans: Hint: - for pipe open at both, adding end correction at both ends we have

\[ V = 2n_1(\ell_1 + 2e) \]  

\[ V = 2n_2(\ell_2 + 2e) \]

Equating equations (1) & (2) we have,

\[ 2n_1(\ell_1 + 2e) = 2n_2(\ell_2 + 2e) \]

\[ \therefore n_1\ell_1 - n_2\ell_2 = 2e(n_2 - n_1) \]

\[ \therefore 2e = \frac{n_1\ell_1 - n_2\ell_2}{n_2 - n_1} \]

\[ \therefore e = \frac{n_1\ell_1 - n_2\ell_2}{2(n_2 - n_1)} \]

****Q Explain free and forced vibration gives their examples.

Ans: - **Free vibration:** - If the body is displaced from its stable equilibrium position & released, the restoring forces acting on the body may set the body into vibration, such vibrations are called as free vibrations & the frequency with which the body vibrates is called natural frequency of the body.

The natural frequency depends upon the dimensions & elastic properties of the vibrating body. e.g.

1. Vibrations of simple pendulum, vibrations of tuning fork are the free vibrations during the free vibrations there is continuously loss of energy due to frictional resistance of the medium. Therefore amplitude of vibrations goes on decreasing & finally the body comes to rest.
2. When prong of tuning fork is struck on rubber pad, the prong vibrate with a single (natural) frequency and amplitude of vibration decreases due to frictional resistance of air and finally it comes to rest.

3. If a stretched string is plucked at some point it performs vibration s with natural frequency.


**Force vibrations:** A body is said to perform forced vibrations, if it is made to vibrate, by an external periodic force at a frequency which is different from the natural frequency of the body.

E.g. vibrations of the table top in contact with vibrating fork, vibrations of the air column in the pipe with vibrating tuning fork are the examples of forced vibrations. The amplitude of the forced vibration depends on the difference between frequency of the external periodic force & the natural frequency of the body. If the difference is large the amplitude of forced vibrations is small. The amplitude of vibrations is large as compared to that of free vibration.

**Distinguish between free vibration and forced vibration:**

<table>
<thead>
<tr>
<th>Free Vibration</th>
<th>Forced Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. These vibrations are produced when the body is displaced from its mean position</td>
<td>1. Forced vibration is produced under the action of external periodic force.</td>
</tr>
<tr>
<td>2. Frequency depends on the dimensions and elastic properties of the body.</td>
<td>2. Frequency depends on nature of external applied periodic force on the body.</td>
</tr>
<tr>
<td>3. Its vibration with a definite frequency, called as natural frequency.</td>
<td>3. It can take place at any frequency other than natural frequency of the body.</td>
</tr>
<tr>
<td>4. Amplitude depends upon extent to which the body is disturbed from its equilibrium position.</td>
<td>4. Amplitude depends upon the difference between the natural frequency of external periodic force.</td>
</tr>
</tbody>
</table>

*Resonance with example:*
The resonance is a special case of forced vibrations in which natural frequency of body becomes exactly equal to the frequency of external periodic force and the body vibrates with maximum amplitude. The phenomenon is known as resonance.

Example of resonance:

i. Two identical pendulums are suspended side by side from a horizontal flexible support (rubber or string) . When pendulum A is set into oscillations it is called driver. The horizontal supports respond this oscillation. It is observed that B also starts vibrating with its amplitude increasing to maximum. (The phase difference of $\pi/2$ is between oscillation of A and B) The response of pendulum B to the forced oscillation depends upon the relative length of two pendulums. Response is inversely proportional to its relative length, the best response (i.e. resonance) is observed at equal lengths of pendulum.

(2) A stethoscope is an acoustic medical device.

Doctors use to listen the sound produced by the heart and other organs of the human body. It is a simple device which merely conducts the desired sound to the doctor's ears and excludes all other irrelevant sounds. It consists of a body contact piece connected by hollow rubber tubes to earpieces, different sized organ pipes resonate at different frequencies; if the contact piece has dimensions close to those needed to resonate at the frequency being picked up, the stethoscope is more sensitive and hence more. Useful to the doctor.

Acoustic stethoscope are familiar to most people and operate on the transmission of sound from the chest piece, via air filled hollow tube to the listener's ears. The chest piece usually consists of two sides that can be placed against the patient, heart beats vibrate the diaphragm, creating acoustic pressure waves which travel up the tube to listener's ears. If the bell is placed on the patient, the vibration of the skin directly produces acoustic pressure waves travelling up the listener's ears. This bell transmits low frequency sounds while the
diaphragms transmits higher' frequency sounds. This two sided stethoscope was invented by Rappaport and Sprague in the early part of 20th century.

(3) Electrical resonance is provided by tuning of radio receiver. At given instant several broadcasting stations are emitting electromagnetic waves of different frequencies. All these are simultaneously intercepted by the aerial/antenna connected to the radio receiver. By tuning the dial marked ‘tuning' the frequency of oscillatory circuit in tube receiver is made equal to the frequency of wave broadcast by the desired station. Thus only the signals of that frequency are amplified by the receiver and signal of other frequencies are rejected.

(4) The resonant vibrations of stretched string and their laws of vibrations can be studied by using (i) Sonometer and (ii) Melde's experiment.

(i) Sonometer experiment demonstrates the formation of stationary wave, resonance and to determine unknown frequency of fork.

\[ N = n = \frac{1}{2L} \sqrt{\frac{T}{m}} \]

Where \( N \) = frequency of the fork, \( n \) = frequency of vibrating string and \( m \) = linear density of string.

(ii) Melde's Experiment

A) Parallel position

\[ N = 2n = \frac{p_1}{\ell} \sqrt{\frac{T}{m}} \]

B) Perpendicular position

\[ N = \frac{p_2}{2\ell} \sqrt{\frac{T}{m}} \]

**The main applications of resonance in everyday life:**

i. By using principle of resonance, unknown frequency of vibrating tuning fork can be calculated.

ii. Radio receiver can be tuned to a desired frequency by using principle of resonance.

iii. In resonance tube when frequency of vibrating tuning fork is unison with the natural frequency of vibration of air column, a loud resonant sound is heard by observer. The length of air column is measured and the velocity of sound in air at room temperature is calculated.

iv. A hollow wooden box is provided for the stringed musical instrument (examples: Violin, guitar, sitar, and sarangi) When the string vibrates, the air inside the box vibrates with same frequency, Resonance occurs and a louder, pleasant sound is produced.
v. To increase the intensity of sound in musical instrument.

*The disadvantage of resonance:*

i. Soldiers are ordered to break their regular stepping in marching. While crossing a suspension bridge. Because the frequency of their step may become equal to natural frequency of the bridge. Thus at resonance of frequency, difference in frequencies becomes zero and amplitude of vibration of bridge becomes maximum hence the bridge may collapse. ii. When the speed of an aircraft increases different parts are forced to vibrate. Resonance is undesirable

iii. If the frequency of clapping of hands by a cheering audience is the same as the natural frequency of the roof, the roof may be brought down due to resonance.

iv. In a rough sea, if the natural frequency of swinging of ship is equal to the frequency of water waves (frequency of external periodic force), the amplitude of swinging of ship may cross its safety limit and it may become dangerous. This can be avoided by changing the speed and direction of ship.

*Musical instruments:*

The musical instruments can be classified in to the four types. (1) Stringed instruments (2) Wind instruments (3) Percussion instruments (4) Solid instruments.

1. **String instrument**, in which sound produced by plunking of strings.
   Examples:
   (a) Pianoforte, santoor are instruments in which string stuck by a hammer.
   (b) Sitar, veena, guitar, tanpura are instruments in which string pluck by a finger.(The violin note and sitar note may have the same frequency, the two notes appear different because of the different number of overtone accompanying them.)

2. **Wind instruments** in which sound is produced by setting vibration of air columns.
   Examples:
   (a) **Flute**: The flute is "simple wind instrument consisting of a cylindrical pipe of either metal or bamboo closed at one end A and open at the other end B. The pipe has a narrow opening M near its closed end, through which air is blown. Seven more holes are drilled along the pipe between M and B. These holes can be closed by fingers of the player. When all holes are closed and air is blown through hole the air column inside the flute is set into vibration. The incident wave (pressure wave) and reflected wave forms longitudinal stationary wave. When any hole is uncovered, the flute acts as a pipe open at both ends. The distances of the seven holes from M are adjusted so that when the hole is uncovered in succession, the notes (sa, re, ga, ma-s-etc.) are produced.
(b) **Bugle**: It is pipe instrument without reeds. When flute is played softly, it produces almost pure sound note free from any overtone. The frequency of the sound produced by a flute (pipe) depends on its length. When holes of flute are opened or closed, its effective length changes and sound of different frequencies are produced.

(c) **Bassoon**: It is pipe instruments with reeds.

(d) **Harmonium**: It is reed instrument without pipe. It is wind musical instrument. It consists of a keyboard in which air is set into vibrations by means of thin metal reeds. Each key is tuned to its appropriate pitch. **Reeds**: A reed is fastened at one end of a block in which there is hole behind the reed can vibrate freely. **Bellows**: When the wind is forced through the aperture underneath a reed and key is pressed down. The blast of air (bellows) sets the reed into vibrations. It produces a fairly loud sound. The major notes of the diatonic scale sa, re, ga, ma, pa, dha, ni, sa are indicated by the white keys and five additional notes in octave by the black key. Generally, there is three and half octave with about 41 reeds. Each 01le of them is controlled by a key.

(3) **Percussion instruments** in which sound produced by setting vibrations in a stretched membrane.

Drums, tabla, mridangam etc, are examples of percussion instrument.

**Important formulas at a glance:**

1. Speed of transverse wave along a string.\[ V = \sqrt{\frac{T}{m}} \]
2. Vibrations of string

   A) Fundamental frequency
   \[ n = \frac{1}{2\ell} \sqrt{\frac{T}{m}} \]

   B) Second harmonics (first overtone)
   \[ n_1 = \frac{1}{\lambda_1} \sqrt{\frac{T}{m}}, \quad n_1 = 2n \]

   C) Third harmonics (second overtone)
   \[ n_2 = \frac{3}{2\ell} \sqrt{\frac{T}{m}}, \quad n_2 = 3n \]

3. Melde’s Experiment
   A) Parallel position
   \[ N = 2n = \frac{p_1}{\ell} \sqrt{\frac{T}{m}} \]
   B) Perpendicular position
   \[ N = \frac{p_2}{2\ell} \sqrt{\frac{T}{m}} \]

   C) Both positions \( TP^2 = \text{constant} \).
   D) Number of loop formed in perpendicular position = 2 x Number of loop in parallel position.
4. Vibration of air column in pipe closed at one end:

A) Fundamental frequency, \( n = \frac{V}{4l} \)

B) Second harmonics (first over tone)

\[ n_1 = \frac{3V}{4l} = 3n \]

C) Third harmonics (second over tone)

\[ n_2 = \frac{5V}{4l} \quad \text{or} \quad n_2 = 5n \]

5. Vibration of air column in pipe open at both the end:

A) Fundamental frequency, \( n = \frac{V}{2\ell} \)

B) Second harmonics (first over tone)

\[ n_1 = \frac{V}{\lambda_1} = \frac{V}{\ell}, \quad n_1 = 2n \]

C) Third harmonics (second over tone)

\[ n_2 = \frac{V}{\lambda_2} = \frac{3V}{2\ell}, \quad n_2 = 3n \]

6. Resonance tube experiment:

A) End correction \( e = 0.3d \)

B) \( V = 4n(l + 0.3d) \)

C) \( V = 2n(\ell_2 - \ell_1) \)

7. Equation of stationary waver

\[ y = A \sin(2\pi nt) \]

where Amplitude of stationary wave

\[ A = 2a \cos\left(2\pi \frac{x}{\lambda}\right) \]

Thanks to everyone