The Theory of Multiplier: Concept, Derivation and Calculation.

The Concept of Multiplier:

The theory of multiplier occupies an important place in the modern theory of income and employment. The concept of multiplier was first of all developed by F.A. Kahn in the early 1930s. But Keynes later further refined it. F.A. Kahn developed the concept of multiplier with reference to the increase in employment, direct as well as indirect, as a result of initial increase in investment and employment. Keynes, however, propounded the concept of multiplier with reference to the increase in total income, direct as well as indirect, as a result of original increase in investment and income. Therefore, whereas Kahn’s multiplier is known as ‘employment multiplier’, Keynes’s multiplier is known as investment or income multiplier. The essence of multiplier is that total increase in income, output or employment is manifold the original increase in investment. For example, if investment equal to Rs. 100 crores is made, then the income will not rise by Rs. 100 crores only but a multiple of it. For this Government will pay wages to the labourers engaged, prices for the materials to the suppliers and remunerations to other factors who make contribution to the work of road-building. The total cost will amount to Rs. 100 crores. This will increase incomes of the people equal to Rs. 100 crores. But this is not all. The people who receive Rs. 100 crores will spend a good part of them on consumer goods. Suppose marginal propensity to consume of the people is 4/5 or 80%. Then out of Rs. 100 crores they will spend Rs. 80 crores on consumer goods, which would increase incomes of those people who supply consumer goods equal to Rs. 80 crores. But those who receive these Rs. 80 crores will also in turn spend these incomes, depending upon their marginal propensity to consume. If their marginal propensity to consume is also 4/5, then they will spend Rs. 64 crores on consumer goods.
This will further increase income of some other people equal to Rs. 64 crores. In this way, the chain of consumption expenditure would continue and the income of the people will go on increasing. But every additional increase in income will be progressively less since a part of the income received will be saved. Thus, we see that the income will not increase by only Rs. 100 crores, which was initially invested in the construction of roads, but by many time more.

**Derivation of Investment Multiplier:**

How much increase in national income will take place as a result of an initial increase in investment can be expressed in the following mathematical form:

Increase in income

Or

\[ \Delta Y = 100 + 100 \times \frac{4}{5} + 100\left(\frac{4}{5}\right)^2 + 100\left(\frac{4}{5}\right)^3 + 100\left(\frac{4}{5}\right)^4 \]

\[ = 100 \left[ 1 + \left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \left(\frac{4}{5}\right)^4 \right] \]

But the above series is one of geometric progression. Therefore, increase in income (\(\Delta Y\))

\[ = 100 \times \frac{1}{1 - \frac{4}{5}} \]

\[ = 100 \times \frac{1}{\frac{1}{5}} \]

\[ = 100 \times 5 \]

\[ = 500 \]

It is thus clear that if the marginal propensity to consume is 4/5, the investment of Rs. 100 crores leads to the increase in the national income by Rs. 500 crores. Therefore, multiplier here is equal to 5. We can express this in a general formula.
if $\Delta Y$ stands for increase in income, $\Delta I$ stands for increase in investment and MPC for marginal propensity to consume, we can write the equation (i) above as follows

$$\Delta Y = \frac{\Delta I}{1 - MPC}$$

$$\Delta Y/\Delta I = 1/1 - MPC$$

$\Delta Y/\Delta I$ = measure size of the multiplier

measure size of the multiplier = $1/1 - MPC$

It is clear from above that the size of multiplier depends upon the marginal propensity to consume of the community. The multiplier is the reciprocal of one minus marginal propensity to consume. However, we can express multiplier in a simpler form. As we know that saving is equal to income minus consumption, one minus marginal propensity to consume will be equal to marginal propensity to save, that is, $1 - MPC = MPS$. Therefore, multiplier is equal to

$$1/1 - MPC = 1/MPS$$

**Algebraic Derivation of Multiplier:**

The multiplier can be derived algebraically as follows:

Writing the equation for the equilibrium level of income we have

$$Y = C + I$$

As in the multiplier analysis we are concerned with changes in income induced by changes in investment, rewriting the equation (1) in terms of changes in the variables we have

$$\Delta Y = \Delta C + \Delta I$$
In the simple Keynesian model of income determination, change in investment is considered to be autonomous or independent of changes in income while changes in consumption are function of changes in income. In the consumption function,

\[ C = a + bY \]

where \( a \) is a constant term, \( b \) is marginal propensity to consume which is also assumed to remain constant therefore, change in consumption can occur only if there is change in income thus

Theory of multiplier

\[ \Delta C = b \Delta Y \]

\[ \Delta Y = b \Delta Y + \Delta I \]

\[ \Delta Y - b \Delta Y = \Delta I \]

\[ \Delta Y(1-b) = \Delta I \]

Or

\[ \Delta Y = \frac{1}{1-b} \Delta I \]

\[ \Delta Y / \Delta I = 1/(1 - b) \]

As \( b \) stands for marginal propensity to consume

\[ \Delta Y / \Delta I = 1/(1 - MPC) = 1/MPS \]

This is the same formula of multiplier as obtained earlier. Note that the value of multiplier \( \Delta Y / \Delta I \) will remain constant as long as marginal propensity to consume remains the same.

**Calculating the Size or Value of Multiplier:**

It follows from above that the size or value of multiplier is the reciprocal of marginal propensity to save. Therefore, we can obtain the value of multiplier if we know the marginal propensity to consume or the marginal propensity to save of the community. Given the size of multiplier form
the net increase in investment, we can find out the total increment in income that will occur as a result of investment.

**If the marginal propensity to consume of a community is equal to 2/3, we can find out the size of multiplier as under:**

Multiplier, \( k = \frac{1}{1-MPC} \)

\[
\frac{1}{1} - \frac{2}{3} = \frac{1}{1/2} = 3
\]

Likewise, if the marginal propensity to consume is equal to ½ or 0.5, then the multiplier:

\[
\frac{1}{1} - \frac{1}{2} = \frac{1}{1/2} = 2
\]

**Two Limiting Cases of the Value of Multiplier:**

There are two limiting cases of the multiplier. One limiting case occurs when the marginal propensity to consume is equal to one, that is, when the whole of the increment in income is consumed and nothing is saved. In this case, the size of multiplier will be equal to infinity, that is, a small increase in investment will bring about a very large increase in income and employment so that full employment is reached and even the process goes beyond that. “In such circumstances, the Government would need to employ only one road builder to raise income indefinitely, causing first full employment and then a limitless spiral of inflation.”

However, this is unlikely to occur since marginal propensity to consume in the real world is less than one. The other limiting case occurs when marginal propensity to consume is equal to zero, that is, when nothing out of the increment in income is consumed, and the whole increment in income is saved.
In this case, the value of the multiplier will be equal to one. That is, in this case, the increment in income will be equal to the original increase in investment and not a multiple of it. But in actual practice the marginal propensity to consume is less than one but more than zero ($1 > \Delta C/\Delta Y > 0$). Therefore, the value of the multiplier is greater than one but less than infinity.

**Diagrammatic Representation of Multiplier:**

We have already explained that the level of national income is determined by the equilibrium between aggregate demand and aggregate supply. In other words, the level of national income is fixed at the level where $C + I$ curve intersects the 45° income curve. With such a diagram we can explain the multiplier. The multiplier is illustrated in Fig. 9.1. In this figure $C$ represents marginal propensity to consume. Marginal propensity to consume has been here assumed to be equal to $1/2$ i.e., 0.5. Therefore, the slope of the curve $C$ of marginal propensity to consume curve $C$ has been taken to be equal to 0.5. $C + I$ represents aggregate demand curve. It will be seen from Fig. 91 that the aggregate demand curve $C + I$ which intersects the 45° line at point E so that the level of income equal to $OY_1$ is determined.
If investment increases by the amount EH we can then find out how much increment in income will occur as a result of this. As a consequence of increase in investment by EH, the aggregate demand curve shifts upward to the new position C + I’. This new aggregate demand curve C + I’ intersects the 45° income line at point F so that the equilibrium level of income increases to OY₂.

Hence as a result of net increase in investment equal to EH, the income has increased by Y₁Y₂. It will be seen from the figure that Y₁Y₂ is greater than EH. On measuring, it will be found that Y₁Y₂ is twice the length of EH. This is as it is expected because the marginal propensity to consume is here equal to 1/2 and therefore the size of multiplier will be equal to 2.

The multiplier can be illustrated through saving-investment diagram also. In a previous chapter we explained the determination of national income also through saving the investment. Therefore, the multiplier can also be explained with the help of saving- investment diagram, as has been shown in Fig. 9.2. In this figure SS is the saving curve indicating that as the level of income increases, the community plans to save more. II is the investment curve showing the level of investment planned to be undertaken by the investors in the community.
the investment has been taken to be a constant amount and autonomous of changes in income. This investment level \( I_0 \) has been determined by marginal efficiency of capital and the rate of interest. Investment being autonomous of income means that it does not change with the level of income. Keynes treated investment as autonomous of income and we will here follow him. It will be seen from Fig. 9.2 that saving and investment curves intersect at point E, that is, planned saving and planned investment are in equilibrium at the level of income \( OY_1 \).

Thus, with the given saving and investment curves level of income equal to \( OY_1 \) is determined. Now suppose that there is an increase in investment by the amount \( II' \). With this increase in investment, the investment curve shifts to the new dotted position \( I'I' \).

This new investment curves \( I'I' \) intersects the saving curve at point F and a new equilibrium as reached at the level of income \( OY_2 \). A glance at the Fig. 9.2 will reveal that the increase in income \( Y_1 Y_2 \) is twice the increase in investment by \( II' \). Thus multiplier is here equal to \( [K=1/0.5=2] \).